# Modern Physics:Lecture 07: Step Potential

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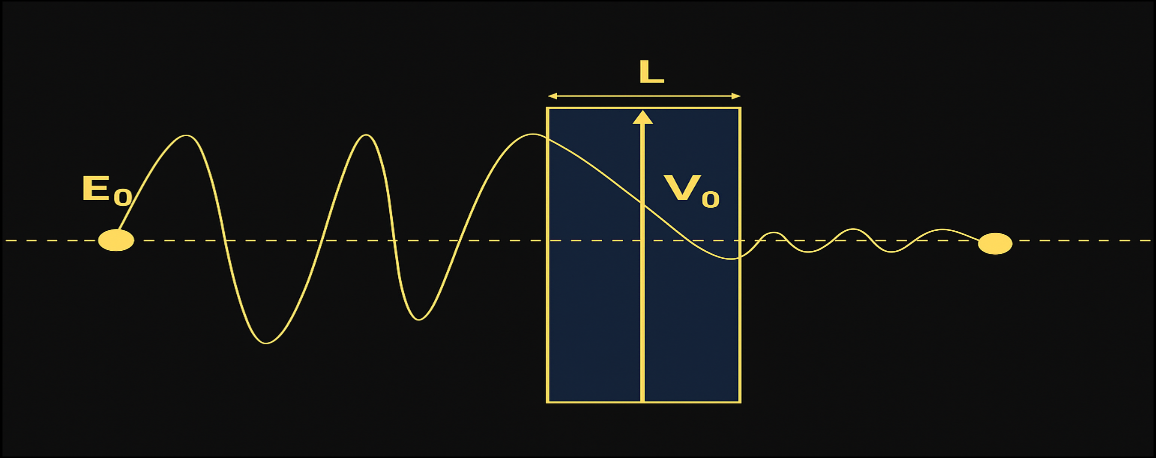
# Outline

# Introduction

- Quantum mechanics reveals phenomena that defy classical intuition.
- One fundamental example is the finite potential barrier, central to:
  - Tunneling phenomena
  - Quantum transport in nanostructures
  - Nuclear and semiconductor physics
- This presentation explores the full analytical treatment of this system.

# particle in a box potential



# Physical Setup: Finite Potential Barrier

$$V(x) = \begin{cases} 0 & x < 0 \\ V_0 & 0 \leq x \leq L \\ 0 & x > L \end{cases}$$

- Particle of energy $E$ is incident from the left.
- Two regimes of interest:
  - $E > V_0$: Over-the-barrier transmission
  - $E < V_0$: Quantum tunneling

# Why Time-Independent Schrödinger Equation?

- We consider stationary states $\psi(x)e^{-iEt/\hbar}$.
- Time-independent Schrödinger equation (TISE) governs $\psi(x)$:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi$$

- Reflects energy eigenstates, sufficient to compute transmission and reflection coefficients.
- Time-dependent effects (e.g. modulated barriers) require separate treatment.

Define:

$$k = \frac{\sqrt{2mE}}{\hbar},$$
$$\kappa = \frac{\sqrt{2m(E - V_0)}}{\hbar} \quad (E > V_0),$$
$$\alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar} \quad (E < V_0)$$

**Regions:**

- **Region I (x ¡ 0):** $\psi_I(x) = Ae^{ikx} + Be^{-ikx}$
- **Region II** $(0 \leq x \leq L) : \psi_{II}(x) = \begin{cases} Ce^{i\kappa x} + De^{-i\kappa x} & E > V_0 \\ Ce^{\alpha x} + De^{-\alpha x} & E < V_0 \end{cases}$

**Region III (x ¿ L):** $\psi_{III}(x) = Fe^{ikx}$

# Continuity Conditions at Boundaries

Wavefunction and its derivative must be continuous at $x = 0$ and $x = L$:

$$\psi_I(0) = \psi_{II}(0), \qquad \psi_I'(0) = \psi_{II}'(0)$$
$$\psi_{II}(L) = \psi_{III}(L), \qquad \psi_{II}'(L) = \psi_{III}'(L)$$

These lead to a system of equations for coefficients $A, B, C, D, F$.

# Transmission and Reflection Coefficients

From boundary conditions, solve for ratios:

$$T = \left|\frac{F}{A}\right|^2, \quad R = \left|\frac{B}{A}\right|^2, \quad T + R = 1$$

For $E > V_0$, transmission probability is:

$$T = \frac{4k^2\kappa^2}{(k^2 - \kappa^2)^2 \sin^2(\kappa L) + 4k^2\kappa^2}$$

- **Resonances:** When $\kappa L = n\pi$, $T = 1$
- Full transmission despite the presence of the barrier

# Quantum Tunneling: $E < V_0$

- In classically forbidden region, wavefunction decays exponentially
- Approximate result for transmission:

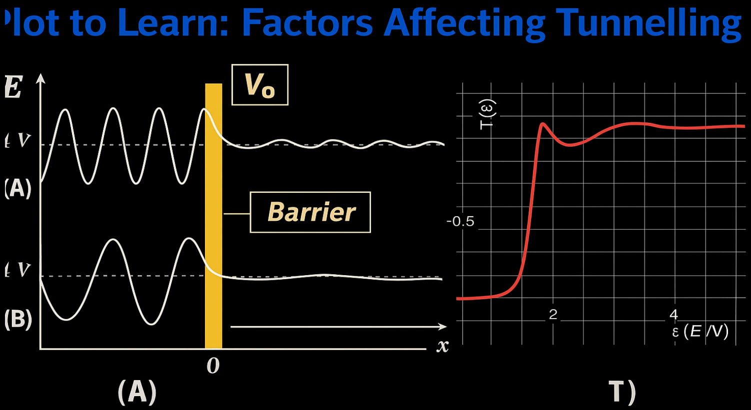
$$T \approx \frac{16E(V_0 - E)}{V_0^2} e^{-2\alpha L}, \quad \alpha = \frac{\sqrt{2m(V_0 - E)}}{\hbar}$$

**Key Features:**

- Exponential sensitivity to barrier width and height
- Enables alpha decay, electron tunneling, etc.

# Energy-Dependent Transmission

- Plot $T(E)$ shows resonances and tunneling suppression
- Design implications for nano-devices



# Numerical Example

**Case: Electron with $E = 2$ eV, $V_0 = 4$ eV, $L = 0.5$ nm**

- Compute:

$$\alpha \approx 1.025 \times 10^{10}\ \mathrm{m}^{-1}, \quad T \approx e^{-2\alpha L} \approx 0.0043$$

- Tunneling probability is small but nonzero

# Applications in Modern Physics

- **STM:** Electrons tunnel across vacuum gap, enabling atomic-scale imaging
- **Nuclear decay:** Alpha particles tunnel through nuclear potential
- **Tunnel diodes:** Exploit negative differential resistance from quantum tunneling

# Summary and Outlook

- Finite barriers illustrate key non-classical phenomena
- Mathematical treatment provides precise predictions for tunneling
- Basis for understanding mesoscopic and quantum device physics
- Extensions: Time-dependent potentials, higher dimensions, multiple barriers

Thank You!

Questions?