1. Resistivity by Four Probe Method
Resistivity by Four Probe Method

**AIM**

To determine the resistivity of semiconductors by Four probe Method.

**APPARATUS**

The experimental setup consists of probe arrangement, sample, oven 0-200°C, constant current generator, oven power supply and digital panel meter (measuring voltage and current).

Four probe apparatus is one of the standard and most widely used apparatus for the measurement of resistivity of semiconductors.

This method is employed when the sample is in the form of a thin wafer, such as a thin semiconductor material deposited on a substrate. The sample is millimeter in size and having a thickness w. It consists of four probe arranged linearly in a straight line at equal distance s from each other. A constant current is passed through the two probes and the potential drop V across the middle two probes is measured. An oven is provided with a heater to heat the sample so that behavior of the sample is studied with increase in temperature.

**THEORY**

At a constant temperature, the resistance, R of a conductor is proportional to its length L and inversely proportional to its area of cross section A.

\[
R = \rho \frac{L}{A} \quad (1)
\]

Where \( \rho \) is the resistivity of the conductor and its unit is ohm meter.

A semiconductor has electrical conductivity intermediate in magnitude between that of a conductor and insulator. Semiconductor differs from metals in their characteristic property of decreasing electrical resistivity with increasing temperature.

According to band theory, the energy levels of semiconductors can be grouped into two bands, valence band and the conduction band. In the presence of an external electric field it is electrons in the valence band that can move freely, thereby responsible for the electrical conductivity of semiconductors. In case of intrinsic semiconductors, the Fermi level lies in between the conduction band minimum and valence band maximum. Since conduction band lies above the Fermi level at 0K, when no thermal excitations are available, the conduction band remains unoccupied. So conduction is not possible at 0K, and resistance is infinite. As temperature increases, the occupancy of conduction band goes up, thereby resulting in decrease of electrical resistivity of semiconductor.

Resistivity of semiconductor by four probe method

1. The resistivity of material is uniform in the area of measurement.
2. If there is a minority carrier injection into the semiconductor by the current-carrying electrodes most of the carriers recombine near electrodes so that their effect on conductivity is negligible.
3. The surface on which the probes rest is flat with no surface leakage.
4. The four probes used for resistivity measurement contact surface at points that lie in a straight line.
5. The diameter of the contact between metallic probes and the semiconductor should be small compared to the distance between the probes.
6. The boundary between the current carrying electrodes and the bulk material is hemispherical and small in diameter.
7. The surface of semiconductor material may be either conducting and non-conducting. A conducting boundary is one on which material of much lower resistivity than semiconductor has been plated. A non-conducting boundary is produced when the surface of the semiconductor is in contact with insulator.

Fig: 2 show the resistivity probes on a die of material. If the side boundaries are adequately far from the probes, the die may be considered to be identical to a slice. For this case of a slice of thickness w and the resistivity is computed as
Resistivity by Four Probe Method

\[ \rho = \frac{\rho_0}{f}\left(\frac{w}{S}\right) \]  \hspace{1cm} (2)

The function, \( f(w/S) \) is a divisor for computing resistivity which depends on the value of \( w \) and \( S \).

We assume that the size of the metal tip is infinitesimal and the sample thickness is greater than the distance between the probes.

\[ \rho_0 = \frac{V}{I} \times \frac{1}{2 \pi S} \]  \hspace{1cm} (3)

Where \( V \) – the potential difference between inner probes in volts.
\( I \) – Current through the outer pair of probes in ampere.
\( S \) – Spacing between the probes in meter.

Temperature dependence of resistivity of semiconductor

Total electrical conductivity of a semiconductor is the sum of the conductivities of the valence band and conduction band carriers. Resistivity is the reciprocal of conductivity and its temperature dependence is given by

\[ \rho = A \exp\left(\frac{E_g}{kT}\right) \]  \hspace{1cm} (4)

Where \( E_g \) – band gap of the material
\( T \) – Temperature in kelvin
\( K \) – Boltzmann constant, \( K = 8.6 \times 10^{-5} \) eV/K

The resistivity of a semiconductor rises exponentially on decreasing the temperature.

Applications

1. Remote sensing areas
2. Resistance thermometers
3. Induction hardening process
4. Accurate geometry factor estimation
5. Characterization of fuel cells bipolar plates
Resistivity by Four Probe Method

Procedure for Simulation

Combo Box and Sliders

- **Select Material** - This is used to select semiconductor material for doing the simulator.
- **Range of Current** - One can choose the range of current for the current source.
- **Current Slider** - It ranges from 1mA to 200mA. (Note: The divisions in the slider is fixed as 100). If 20mA current is selected in the combo box, the slider value will range from 0mA to 20mA, with an interval of 0.2mA and if the value is 200mA in the combo box, slider value changes from 0mA to 200mA with an interval of 2mA.
- **Range of oven** - This combo box is used to fix the temperature to a particular range.
- **Oven** - Oven is used to vary the temperature up to 200°C.
  - **Set Button** – It is used to fix the temperature in the oven.
  - **Run Button** – After setting the temperature, using run button we can start heating the oven.
  - **Wait Button** – It is used to stop heating the oven at a particular temperature.
  - **Measure Button** – It is used to display the present temperature of the oven.
- **Select Range Combo Box** – Options are X1 and X10.
- **Temperature slider** - It ranges from 27°C to 200°C. active only by clicking the Set button and become inactive after clicking Run button. If X1 is in combo box, the slider value ranges from 27°C to 99°C and If the value is X10 in combo box, slider value changes from 2.7°C to 200°C.
- **Voltmeter Combo Box** - Options are 1 mV, 10 mV, 100 mV, 1 V, 10 V. One can select it for getting output in a particular range.

Procedure

1. Select the semiconductor material from the combo box.
2. Select the source current from the slider. Restrict the slider based on the range of current.
3. Select the Range of oven from the combo box.
4. Set the temperature from the slider.
5. Click on the Run Button to start heating the oven in a particular interval, from the default 25°C to the temperature that we set already.
6. Click on the Wait button to stop heating.
7. Click on the Set button to display the temperature that we set in the oven.
8. Click on the Measure button to display the present temperature in the oven.
9. Measure the Voltage using Voltmeter.
10. Calculate the Resistivity of semiconductor in eV for the given temperature using equation (2) and (3).
11. A Graph is plotted with Temperature along x-axis and resistivity of semiconductor along y-axis.

Procedure for Real Lab

In real lab, four probes are placed on the sample as shown in Fig:1. Connections are made as shown in the simulator. A constant current is passed through the outer probes by connecting it to the constant current source of the set up. The current is set to 8mA. The voltage developed across the middle two probes is measured using a digital milli-voltmeter. The trial is repeated by placing the four probe arrangement inside the oven. The oven is connected to the heater supply of the set up. For different temperatures, up to 150°C, the voltage developed is noted and tabulated.

The distance between the probes(S) and the thickness of the crystal (W) are measured. The values of (W/S) are calculated and the value of the function f(W/S) is taken from the standard table. Using equation (2) and (3), calculate ρ for various temperatures.

Observations and Calculations
Resistivity can be calculated by using the equation given below.
Here we take,
Distance between the probes, $S$ as 0.2 cm and
Thickness of the sample, $w$ as 0.05 cm.
From standard table $f(w/S) = 5.89$
\[
\rho = \frac{R_0}{f\left(\frac{w}{S}\right)} = \text{.................., Ohm cm}
\]
\[
\rho_0 = \frac{V}{I} \times 2\pi w^2 = \text{.................., Ohm cm}
\]

Result

The resistivity of the given semiconductor by Four probe Method = \text{..........................Ohm cm}
2. Hall effect experiment: Determination of charge carrier density
Aim:
1. To determine the Hall voltage developed across the sample material.
2. To calculate the Hall coefficient and the carrier concentration of the sample material.

Apparatus:
Two solenoids, Constant current supply, Four probe, Digital gauss meter, Hall effect apparatus (which consist of Constant Current Generator (CCG), digital milli voltmeter and Hall probe).

Theory:
If a current carrying conductor placed in a perpendicular magnetic field, a potential difference will generate in the conductor which is perpendicular to both magnetic field and current. This phenomenon is called Hall Effect. In solid state physics, Hall effect is an important tool to characterize the materials especially semiconductors. It directly determines both the sign and density of charge carriers in a given sample.

Consider a rectangular conductor of thickness $t$ kept in XY plane. An electric field is applied in X-direction using Constant Current Generator (CCG), so that current $I$ flow through the sample. If $w$ is the width of the sample and $t$ is the thickness. There for current density is given by

$$J_x = \frac{I}{wt} \quad (1)$$

![Fig.1 Schematic representation of Hall Effect in a conductor.](https://vlab.amrita.edu/?sub=1&brch=282&sim=879&cnt=1)

CCG – Constant Current Generator, $J_x$ – current density
$\vec{e}$ – electron, $B$ – applied magnetic field
$t$ – thickness, $w$ – width
$V_H$ – Hall voltage

If the magnetic field is applied along negative z-axis, the Lorentz force moves the charge carriers (say electrons) toward the y-direction. This results in accumulation of charge carriers at the top edge of the sample. This set up a transverse electric field $E_y$ in the sample. This develop a potential difference along y-axis is known as Hall voltage $V_H$ and this effect is called Hall Effect.

A current is made to flow through the sample material and the voltage difference between its top and bottom is measured using a voltmeter. When the applied magnetic field $B=0$, the voltage difference will be zero.

We know that a current flows in response to an applied electric field with its direction as conventional and it is either due to the flow of holes in the direction of current or the movement of electrons backward. In both cases, under the application of magnetic field the magnetic Lorentz force, $F_m = q(\vec{v} \times \vec{B})$ causes the carriers to curve upwards. Since the charges cannot escape from the material, a vertical charge imbalance builds up. This charge imbalance produces an electric field which counteracts with the magnetic force and a steady state is established. The vertical electric field can be measured as a transverse voltage difference using a voltmeter.

In steady state condition, the magnetic force is balanced by the electric force. Mathematically we can express it as

$$eE = evB \quad (2)$$

Where 'e' the electric charge, 'E' the hall electric field developed, 'B' the applied magnetic field and 'v' is the drift velocity of charge carriers. And the current 'I' can be expressed as,
Where 'n' is the number density of electrons in the conductor of length 'l', breadth 'w' and thickness 't'.

Using (1) and (2) the Hall voltage $V_H$ can be written as,

\[ V_H = E_w = vBw = \frac{IB}{ne} \] 

\[ V_H = R_H \frac{IB}{t} \]  \hspace{1cm} (4)

by rearranging eq(4) we get

\[ R_H = \frac{V_H}{\frac{IB}{t}} \]  \hspace{1cm} (5)

Where $R_H$ is called the Hall coefficient.

\[ R_H = \frac{1}{ne} \]  \hspace{1cm} (6)
**Procedure:**

**Controls**

**Combo box**

- **Select procedure:** This is used to select the part of the experiment to perform.
  1) Magnetic field Vs Current.
  2) Hall effect setup.

- **Select Material:** This slider activate only if Hall Effect setup is selected. And this is used to select the material for finding Hall coefficient and carrier concentration.

**Button**

- **Insert Probe/ Remove Probe:** This button used to insert/remove the probe in between the solenoid.
- **Show Voltage/ Current:** This will activate only if Hall Effect setup selected and it used to display the Hall voltage/ current in the digital meter.
- **Reset:** This button is used to repeat the experiment.

**Slider**

- **Current:** This slider used to vary the current flowing through the Solenoid.
- **Hall Current:** This slider used to change the hall current
- **Thickness:** This slider used to change the thickness of the material selected.

**Procedure for doing the simulation:**

**To measure the magnetic field generated in the solenoid**

- Select **Magnetic field Vs Current** from the procedure combo-box.

- Click **Insert Probe** button

- Placing the probe in between the solenoid by clicking the wooden stand in the simulator.

- Using Current slider, varying the current through the solenoid and corresponding magnetic field is to be noted from Gauss meter.

<table>
<thead>
<tr>
<th>Trial No.</th>
<th>Current through solenoid</th>
<th>Magnetic field generated</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>2</td>
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<td>3</td>
<td></td>
<td></td>
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<td>4</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Hall Effect apparatus**

- Select **Hall Effect Setup** from the Select the procedure combo box

- Click **Insert Hall Probe** button

- Placing the probe in between the solenoid by clicking the wooden stand in the simulator.

- Set "current slider" value to minimum.

- Select the material from "Select Material" combo-box.

- Select the Thickness of the material using the slider **Thickness**.

- Vary the Hall current using the slider **Hall current**.

- Note down the corresponding Hall voltage by clicking "show voltage" button.

- Then calculate Hall coefficient and carrier concentration of that material using the equation

\[ R_H = \frac{V_H}{I \times B} \]

Where \( R_H \) is the Hall coefficient

\[ R_H = \frac{1}{n \varepsilon} \]

And \( n \) is the carrier concentration

- Repeat the experiment with different magnetic file.
**Hall effect experiment:** Determination of charge carrier density

**Procedure for doing real lab**
- Connect ‘Constant current source’ to the solenoids.
- Four probe is connected to the Gauss meter and placed at the middle of the two solenoids.
- Switch ON the Gauss meter and Constant current source.
- Vary the current through the solenoid from 1A to 5A with the interval of 0.5A, and note the corresponding Gauss meter readings.
- Switch OFF the Gauss meter and constant current source and turn the knob of constant current source towards minimum current.
- Fix the Hall probe on a wooden stand. Connect green wires to Constant Current Generator and connect red wires to milli voltmeter in the Hall Effect apparatus.
- Replace the Four probe with Hall probe and place the sample material at the middle of the two solenoids.
- Switch ON the constant current source and CCG.
- Carefully increase the current I from CCG and measure the corresponding Hall voltage $V_H$. Repeat this step for different magnetic field $B$.
- Thickness $t$ of the sample is measured using screw gauge.
- Hence calculate the Hall coefficient $R_H$ using the equation 5.
- Then calculate the carrier concentration $n$. using equation 6.

**Result**

| Hall coefficient of the material | = .................. |
| Carrier concentration of the material | = .................. m$^{-3}$ |
3. Spectrometer, Refractive Index of the material of a prism
Aim

- To determine the refractive index of the material of a prism.

Apparatus

Spectrometer, prism, prism clamp, sodium vapour lamp, lens.

Principle

When a beam of light strikes on the surface of transparent material (Glass, water, quartz crystal, etc.), the portion of the light is transmitted and other portion is reflected. The transmitted light ray has small deviation of the path from the incident angle. This is called refraction.

Refraction is due to the change in speed of light while passing through the medium. It is given by Snell's Law.

\[ \frac{\sin(i)}{\sin(r)} = \frac{n_2}{n_1} \]  \hspace{1cm} \text{(1)}

Where \( i \) is the angle of incident and \( r \) is the angle of refraction. And \( n_1 \) is the refractive index of the first face and \( n_2 \) is the refractive index of the second face.

And the speed of light on both face is related to the equation

\[ \frac{c_1}{c_2} = \frac{n_2}{n_1} \]  \hspace{1cm} \text{(2)}

\( c_1 \) is the velocity of wave in first face and \( c_2 \) is the velocity of wave in second face.

The above figure illustrate the change in refracted angle with respect to the refractive index.

Refractive index of the material of prism

The refractive index of the material of the prism can be calculated by the equation.

\[ n = \frac{\sin\left(\frac{A + D}{2}\right)}{\sin\left(\frac{A}{2}\right)} \]  \hspace{1cm} \text{(3)}

Where, \( D \) is the angle of minimum deviation, here \( D \) is different for different colour.
Spectrometer, Refractive Index of the material of a prism

Controls

Switches
- **Switch On/Off Light**: Used to switch on/off the light.
- **Place Prism/Remove Prism**: This switch used to place the prism on the prism table or remove prism from the prism table.

Slider
- **Slit focus**: This slider used to focus the slit while looking through telescope.
- **Slit width**: Using this slider, width of the slit can be adjusted.
- **Telescope**: Using this slider one can move the telescope from its position.
- **Vernier Table**: Vernier table can be rotate using this slider.

Fine Angle adjustment
- **Telescope**: This is used to fine adjust the telescope.
- **Vernier Table**: Using this slider, we can rotate fine angle.

Measurements
- Here we get the zoomed view of vernier I and II by placing mouse pointer over it.

Procedure:

**Preliminary adjustments:**

Performing simulator
- Focus Telescope on distant object.
- When focus is correct, start button is activated. Then click Start button.
- Switch on the light by clicking Switch On Light button.
- Focus the slit using Slit focus slider.
- Adjust the slit width using Slit width slider.
- Coincide the slit with cross wire in the telescope.

Performing Real Lab:
- 1. Turn the telescope towards the white wall or screen and looking through eye-piece, adjust its position till the cross wires are clearly seen.
- 2. Turn the telescope towards window, focus the telescope to a long distant object.
- 3. Place the telescope parallel to collimator.
- 4. Place the collimator directed towards sodium vapor lamp. Switch on the lamp.
- 5. Focus collimator slit using collimator focusing adjustment.
- 6. Adjust the collimator slit width.
- 7. Place prism table, note that the surface of the table is just below the level of telescope and collimator.
- 8. Place spirit level on prism table. Adjust the base leveling screw till the bubble come at the centre of spirit level.
- 9. Clamp the prism holder.
- 10. Clamp the prism in which the sharp edge is facing towards the collimator, and base of the prism is at the clamp.

Least Count of Spectrometer

One main scale division (N) = ................minute
Number of divisions on vernier (v) = ..............

\[
L.C = \frac{N}{v} = ......................minute
\]

To determine the angle of the Prism:

Performing simulator
- Click Place Prism button.
- Place the edge of prism, pointed towards collimator.
- Move the telescope using Telescope slider up to see the slit on side. Make coincide the slit with the cross wire using fine angle adjusting slider. Then note the reading in the tabular column.
- Move the telescope in the opposite direction and do the same.
- Find the difference between two angle ie 2θ. Hence, find the angle of prism i.e θ.

Performing Real Lab:
- 1. Prism table is rotated in which the sharp edge of the prism is facing towards the collimator.
- 2. Rotate the telescope in one direction up to which the reflected ray is shown through the telescope.
3. Note corresponding main scale and vernier scale reading in both vernier (vernier I and vernier II).
4. Rotate the telescope in opposite direction to view the reflected image of the collimator from the second face of prism.
5. Note corresponding main scale and vernier scale reading in both vernier (vernier I and vernier II).
6. Find the difference between two readings, i.e. \( \theta \)
7. Angle of prism, \( A = \frac{\theta}{2} \)

<table>
<thead>
<tr>
<th>Reading of reflected ray from</th>
<th>Vernier 1</th>
<th>Vernier 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>face 1 (say a)</td>
<td>MSR</td>
<td>VSR</td>
</tr>
<tr>
<td>face 2 (say b)</td>
<td>MSR</td>
<td>VSR</td>
</tr>
<tr>
<td>Difference between a &amp; b</td>
<td>MSR</td>
<td>VSR</td>
</tr>
</tbody>
</table>

Mean \( \bar{\theta} \) = \ldots Degrees

Angle of prism = \ldots Degrees

To determine the Angle of minimum deviation:

Performing simulator

- Rotate the vernier table so as to fall the light from the collimator to one face of the prism and emerged through another face. (refer the given figure).
- Turn the telescope to make the slit coincide with telescope cross wire.
- Slowly rotate the vernier table by using vernier fine adjusting slider.
- Note the position where the slit is stationary for some moment.
- Using telescope fine adjusting slider, make coincide the slit with cross wire.
- Note the reading of vernier 1 and vernier 2.
- Then remove the prism using the button "Remove Prism".
- Carefully turn the telescope so as to get the direct ray from collimator, make it coincide with cross wire in the telescope and again note vernier 1 and 2 readings.
- Hence calculate the angle of minimum deviation (D) by measuring the difference between emerged ray readings and direct ray readings.

Performing Real Lab

1. Rotate the vernier table so that the light from the collimator falling on one of the face of the prism and emerges through the other face.
2. The telescope is turned to view the refracted image of the slit on the other face.
3. The vernier table is slowly turned in such a direction that the image of slit is move directed towards the directed ray; i.e., in the direction of decreasing angle of deviation.
4. It will be found that at a certain position, the image is stationary for some moment. Vernier table is fixed at the position where the image remains stationary.
5. Note the readings on main scale and vernier scale.
6. Carefully remove the prism from the prism table.
7. Turn the telescope parallel to collimator, and note the direct ray readings.
8. Find the difference between the direct ray readings and deviated readings. This angle is called angle of minimum deviation (D).

Refractive index of the material of the prism is determined by using equation (3)

**Result:**

\[
\text{Angle of the Prism} = \ldots \text{Degrees} \\
\text{Angle of minimum deviation of the prism} = \ldots \text{Degrees} \\
\text{Refractive index of the material of the prism} = \ldots
\]
4. Laser beam divergence and spot size
Aim:

To calculate the beam divergence and spot size of the given laser beam.

Laser:

The term LASER is the acronym for Light Amplification by Stimulated Emission of Radiation. It is a mechanism for emitting electromagnetic radiation via the process of stimulated emission. The laser was the first device capable of amplifying light waves themselves. The emitted laser light is a spatially coherent, narrow low-divergence beam. When the waves (or photons) of a beam of light have the same frequency, phase and direction, it is said to be coherent. There are lasers that emit a broad spectrum of light, or emit different wavelengths of light simultaneously. According to the encyclopedia of laser physics and technology, beam divergence of a laser beam is a measure for how fast the beam expands far from the beam waist. A laser beam with a narrow beam divergence is greatly used to make laser pointer devices. Generally, the beam divergence of laser beam is measured using beam profiler.

Lasers usually emit beams with a Gaussian profile. A Gaussian beam is a beam of electromagnetic radiation whose transverse electric field and intensity (irradiance) distributions are described by Gaussian functions.

For a Gaussian beam, the amplitude of the complex electric field is given by

\[ E(r, z) = E_0 \frac{w_0}{w(z)} \exp(-\frac{r^2}{w^2(z)}) \exp(-ikz - i\zeta(z)) \]

where,
- \( r \) - radial distance from the centre axis of the beam
- \( z \) - axial distance from the beam's narrowest point
- \( i \) - imaginary unit (for which \( i^2 = -1 \))
- \( k \) - wave number (in radians per meter).
- \( w_0 \) - radius at which the field amplitude drops to \( 1/e \) and field intensity to \( 1/e^2 \) of their axial values, respectively.
- \( w(z) \) - waist size.
- \( E_0 = |E(0,0)| \)
- \( R_0 \) - radius of curvature of the beam's wavefronts
- \( \zeta(z) \) - Gouy phase shift. It is an extra contribution to the phase that is seen in beams which obey Gaussian profiles.

The corresponding time-averaged intensity (or irradiance) distribution is

\[ I(r, z) = \frac{|E(r, z)|^2}{2\eta} = I_0 \left( \frac{w_0}{w(z)} \right)^2 \exp(-\frac{2r^2}{w^2(z)}) \]

where \( I_0 = I(0,0) \) is the intensity at the center of the beam at its waist. The constant \( \eta \) is defined as the characteristic impedance of the medium through which the beam is propagating.

For vacuum,

\[ \eta = \eta_0 = 377 \text{ ohm} \]

Beam parameters:

Beam parameters govern the behaviour and geometry of a Gaussian beam. The important beam parameters are described below.

Beam divergence:

The light emitted by a laser is confined to a rather narrow cone. But, when the beam propagates outward, it slowly diverges or fans out. For an electromagnetic beam, beam divergence is the angular measure of the increase in the radius or diameter with distance from the optical aperture as the beam emerges.

The divergence of a laser beam can be calculated if the beam diameter \( d_1 \) and \( d_2 \) at two separate distances are known. Let \( z_1 \) and \( z_2 \) are the distances along the laser axis, from the end of the laser to points "1" and "2".
Usually, divergence angle is taken as the full angle of opening of the beam. Then,

\[ \Theta = \frac{d_2 - d_1}{z_2 - z_1} \]

Half of the divergence angle can be calculated as

\[ \Theta_s = \frac{w_2 - w_1}{z_2 - z_1} \]

where \( w_1 \) and \( w_2 \) are the radii of the beam at \( z_1 \) and \( z_2 \).

Like all electromagnetic beams, lasers are subject to divergence, which is measured in milliradians (mrad) or degrees. For many applications, a lower-divergence beam is preferable.

Spot size:

Spot size is nothing but the radius of the beam itself. The irradiance of the beam decreases gradually at the edges.

The distance across the center of the beam for which the irradiance (intensity) equals \( 1/e^2 \) of the maximum irradiance (\( 1/e^2 = 0.135 \)) is defined as the beam diameter. The spot size \( (w) \) of the beam is defined as the radial distance (radius) from the center point of maximum irradiance to the \( 1/e^2 \) point.

Gaussian laser beams are said to be diffraction limited when their radial beam divergence is close to the minimum possible value, which is given by

\[ \Theta = \frac{\lambda}{2 \pi w_0} \]

where \( \lambda \) is the wavelength of the given laser and \( w_0 \) is the radius of the beam at the narrowest point, which is termed as the beam waist.
Performing the real lab:

- Arrange the laser and detector in an optical bench arrangement.
- The laser is switched on and is made to incident on the photodiode.
- Fix the distance, $z$ between the detector and the laser source.
- By adjusting the micrometer of the detector, move the spot in the horizontal direction, from left to right.
- Note the output current for each distance, $x$ from the measuring device.
- Then the beam profile is plotted with the micrometer distance along the X-axis and intensity of current along Y-axis. We will get a gaussian curve (see Fig.1).

![Fig.1 beam profile of Laser beam divergence.](image)

- The experiment is repeated for different detector distances.
- Note the points in the graph where the intensity equals $1/e^2$ of the maximum intensity, say it as $I_e$ (see Fig.1).
- Find the micrometer distance across the beam corresponding to these points (B-A from the Fig.1) for a pair of detector distances $z_1$ and $z_2$. Half of this distance is noted as $w_1$ and $w_2$.
- Then the divergence and spot size of the laser beam can be calculated from the equations.

### Observation and calculation

To find the Least Count of Screw gauge

One pitchscale division ($n$) = .............. mm
Number of divisions on head scale ($m$) = ..............
Least Count (L.C) = $n/m$ = ......................

$z_1 = ......................$ cm  $z_2 = ...............$ cm
Laser beam divergence and spot size

<table>
<thead>
<tr>
<th>P.S.R</th>
<th>H.S.R</th>
<th>Total</th>
<th>mA</th>
</tr>
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<tbody>
<tr>
<td></td>
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1/e² of maximum intensity, Iₑ = ............... mA
Diameter of the beam corresponds to Iₑ, d₁ = ................. mm

<table>
<thead>
<tr>
<th>Distance X in mm</th>
<th>P.S.R</th>
<th>H.S.R</th>
<th>Total</th>
<th>mA</th>
</tr>
</thead>
<tbody>
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</table>

1/e² of maximum intensity, Iₑ = ............... mA
Diameter of the beam corresponds to Iₑ, d₂ = ................. mm

Divergence angle(θ) = (d₂-d₁)/(z₂-z₁) = ................. mrad

Performing the simulator:

- The experimental arrangement is shown in the simulator. A side view and top view of the set up is given in the inset.
- The **start** button enables the user to start the experiment.
- From the combo box, select the desired laser source.
- Then fix a detector distance, say 100 cm, using the slider **Detector distance, z**.
- The z distance can be varied from 50 cm to 200 cm.
- For a particular z distance, change the detector distance x, from minimum to maximum, using the slider **Detector distance, x**. The micrometer distances and the corresponding output currents are noted. The x distances can be read from the zoomed view of the micrometer and the current can be note from the digital display of the output device.
- Draw the graph and calculate the beam divergence and spot size using the steps given above.
- **Show graph** button enables the user to view the beam profile.
- Using the option **Show result**, one can verify the result obtained after doing the experiment.
5. Compound Pendulum- Symmetric
Aim:

To determine:

- The acceleration \( g \) of gravity using a compound pendulum.
- The radius of gyration \( k_O \) of the compound pendulum about an axis perpendicular to the plane of oscillation and passing through its centre of mass.
- The moment of inertia \( I_O \) of the compound pendulum about an axis perpendicular to the plane of oscillation and passing through its centre of mass.

Theory:

In Fig. 1, \( O \) is the point of suspension of the compound pendulum and \( G \) is its centre of mass; we consider the force of gravity to be acting at \( G \). If \( h \) is the distance from \( O \) to \( G \), the equation of motion of the compound pendulum is

\[
I_O \ddot{\theta} = -Mgh \sin \theta
\]

Where \( I_O \) is the moment of inertia of the compound pendulum about the point \( O \).

Comparing to the equation of motion for a simple pendulum

\[
Ml^2 \ddot{\theta} = -MgL \sin \theta
\]

We see that the two equations of motion are the same if we take

\[
\frac{Mgh}{I_O} = \frac{g}{l}
\]

(1)

It is convenient to define the radius of gyration \( k_O \) of the compound pendulum such that if all the mass \( M \) were at a distance \( k_O \) from \( O \), the moment of inertia about \( O \) would be \( I_O \), which we do by writing \( I_O = Mk_O^2 \).

Substituting this into (1) gives us

\[
k_O^2 = h!
\]

(2)

The point \( O' \), a distance \( l \) from \( O \) along a line through \( G \), is called the center of oscillation. Let \( h' \) be the distance from \( G \) to \( O' \), so that \( l = h + h' \). Substituting this into (2), we have

\[
k_O^2 = h' \ell = \ell^2 + h^2 + h'^2
\]

(3)

If \( I_G \) is the moment of inertia of the compound pendulum about its centre of mass, we can also define the radius of gyration \( k_G \) about the centre of mass by writing \( I_G = Mk_G^2 \).

The parallel axis theorem gives us

\[
k_G^2 = h'^2 + k_O^2
\]

Comparing to (3), we have,

\[
k_g = \sqrt{h'h}
\]

(4)

If we switch \( h \) with \( h' \), equation (4) doesn’t change, so we could have derived it by suspending the pendulum from \( O' \). In that case, the center of oscillation would be at \( O' \) and the equivalent simple pendulum would have the same length \( l \). Therefore the period would be the same as when suspended from \( O \). Thus if we know the location of \( G \), by measuring the period \( T \) with suspension at \( O \) and at various points along the extended line from \( O \) to \( G \), we can find \( O' \) and thus \( h' \).

Then using equation (4), we can calculate \( k_O \) and \( I_G = Mk_G^2 \).

Knowing \( h' \) gives us \( l = h + h' \), and since for small angle oscillations the period

\[
T = 2\pi \sqrt{\frac{I}{IG}}
\]

We can calculate \( g \) using
The minimum period $T_{\text{min}}$, corresponds to the minimum value of $l$. Recall that $l = h + h'$ and that $k_G^2/hh'$ is a constant, depending only on the physical characteristics of the pendulum.

Thus, $l = h + k_G^2/h$, and the minimum $l$ occurs when,

$$\frac{\partial^2 l}{\partial h^2} = 1 - \frac{k_G^2}{h^2} = 0$$

i.e., when $h^2 = k_G^2$, $h = h'$ and $l = 2h = 2k_G$.

Thus, at $T_{\text{min}}$, $l = 2k_G$. 

$$g = \frac{4\pi^2 l}{T^2} \tag{5}$$
Performing the real lab:

- The compound bar pendulum AB is suspended by passing a knife edge through the first hole at the end A. The pendulum is pulled aside through a small angle and released, whereupon it oscillates in a vertical plane with a small amplitude. The time for 10 oscillations is measured. From this the period $T$ of oscillation of the pendulum is determined.

- In a similar manner, periods of oscillation are determined by suspending the pendulum through the remaining holes on the same side of the centre of mass $G$ of the bar. The bar is then inverted and periods of oscillation are determined by suspending the pendulum through all the holes on the opposite side of $G$. The distances $d$ of the top edges of different holes from the end A of the bar are measured for each hole. The position of the centre of mass of the bar is found by balancing the bar horizontally on a knife edge. The mass $M$ of the pendulum is determined by weighing the bar with an accurate scale or balance.

- A graph is drawn with the distance $d$ of the various holes from the end A along the X-axis and the period $T$ of the pendulum at these holes along the Y-axis. The graph has two branches, which are symmetrical about $G$. To determine the length of the equivalent simple pendulum corresponding to any period, a straight line is drawn parallel to the X-axis from a given period $T$ on the Y-axis, cutting the graph at four points A, B, C, D. The distances AC and BD, determined from the graph, are equal to the corresponding length $l$. The average length $l = (AC + BD)/2$ and $l/T^2$ are calculated. In a similar way, $l/T^2$ is calculated for different periods by drawing lines parallel to the X-axis from the corresponding values of $T$ along the Y-axis. $l/T^2$ should be constant over all periods $T$, so the average over all suspension points is taken. Finally, the acceleration due to gravity is calculated from the equation $g = 4\pi^2(l/T^2)$.

- $T_{min}$ is where the tangent $EF$ to the two branches of the graph crosses the Y-axis. At $T_{min}$, the distance $EF = l = 2k_G$ can be determined, which gives us $k_G$, the radius of gyration of the pendulum about its centre of mass, and one more value of $g$, from $g = 4\pi^2(2k_G/T_{min}^2)$.

- $k_G$ can also be determined as follows. A line is drawn parallel to the Y-axis from the point $G$ corresponding to the centre of mass on the X-axis, crossing the line ABCD at P. The distances $AP = PD = AD/2 = h$ and $BP = PC = BC/2 = h'$ are obtained from the graph. The radius of gyration $k_G$ about the centre of mass of the bar is then determined by equation (4). The average value of $k_G$ over the different measured periods $T$ is taken, and the moment of inertia of the bar about a perpendicular axis through its centre of mass is calculated using the equation $I_G = M k_G^2$.

Performing the simulation:

- Suspend the pendulum in the first hole by choosing the length 5 cm on the length slider.

- Click on the lower end of the pendulum, drag it to one side through a small angle and release it. The pendulum will begin to oscillate from side to side.

- Repeat the process by suspending the pendulum from the remaining holes by choosing the corresponding lengths on the length slider.

- Draw a graph by plotting distance $d$ along the X-axis and time period $T$ along the Y-axis. (A spreadsheet like Excel can be very helpful here.)

- Calculate the average value of $l/T^2$ for the various choices of $T$, and then calculate $g$ as in step 2 above.
**Compound Pendulum - Symmetric**

- Repeat the experiment in different gravitational environments by selecting an environment from the drop-down environment menu. If the pendulum has been oscillating, press the **Stop** button to activate the environment menu.

**Observations:**

To draw graph:

<table>
<thead>
<tr>
<th>No. of holes from A</th>
<th>Distance of knife edge from A (cm)</th>
<th>Time for 10 oscillations (s)</th>
<th>Time period T (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>Mean (s)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

To find the value of 'g':

<table>
<thead>
<tr>
<th>SI. No</th>
<th>Length of equivalent simple pendulum (cm)</th>
<th>Time period, T (s)</th>
<th>$g$ (cm/s²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>AC (cm)</td>
<td>BD (cm)</td>
<td>Mean l</td>
</tr>
</tbody>
</table>

To find the radius of gyration and the acceleration of gravity (step 3 above):

Radius of gyration about the centre of mass $k_G = \frac{EF}{2} =$ .................

Acceleration of gravity, $g = 4\pi^2\left(\frac{2k_G}{T_{min}}\right)^2 =$ ..................

To find the radius of gyration (step 4 above):

<table>
<thead>
<tr>
<th>SI. No</th>
<th>$h = AD/2$</th>
<th>$h' = BC/2$</th>
<th>$k_G = (h' h)^{1/2}$</th>
</tr>
</thead>
</table>

**Results:**

Average acceleration of gravity, $g = 4\pi^2\left(\frac{M}{T^2}\right) = $ ................. m/s²

Average radius of gyration of the pendulum about its centre of mass, $k_G = $ .................... m

Mass of the pendulum $M =$ .................. Kg

Moment of inertia of the pendulum about its centre of mass, $I_G = k_G^2 =$ ................ Kg m²
6. Kater's pendulum
Aim

To determine $g$, the acceleration of gravity at a particular location.

Apparatus

Kater’s pendulum, stopwatch, meter scale and knife edges.

Theory

Kater’s pendulum, shown in Fig. 1, is a physical pendulum composed of a metal rod 1.20 m in length, upon which are mounted a sliding metal weight $W_1$, a sliding wooden weight $W_2$, a small sliding metal cylinder $w$, and two sliding knife edges $K_1$ and $K_2$ that face each other. Each of the sliding objects can be clamped in place on the rod. The pendulum can suspended and set swinging by resting either knife edge on a flat, level surface. The wooden weight $W_2$ is the same size and shape as the metal weight $W_1$. Its function is to provide as near equal air resistance to swinging as possible in either suspension, which happens if $W_1$ and $W_2$, and separately $K_1$ and $K_2$, are constrained to be equidistant from the ends of the metal rod. The centre of mass $G$ can be located by balancing the pendulum on an external knife edge. Due to the difference in mass between the metal and wooden weights $W_1$ and $W_2$, $G$ is not at the centre of the rod, and the distances $h_1$ and $h_2$ from $G$ to the suspension points $O_1$ and $O_2$ at the knife edges $K_1$ and $K_2$ are not equal. Fine adjustments in the position of $G$, and thus in $h_1$ and $h_2$, can be made by moving the small metal cylinder $w$.

In Fig. 1, we consider the force of gravity to be acting at $G$. If $h_i$ is the distance to $G$ from the suspension point $O_i$ at the knife edge $K_i$, the equation of motion of the pendulum is

$$I_i \ddot{\theta} = -Mg h_i \sin \theta$$

where $I_i$ is the moment of inertia of the pendulum about the suspension point $O_i$, and $i$ can be 1 or 2. Comparing to the equation of motion for a simple pendulum

$$Ml_i \ddot{\theta} = -Mg \sin \theta$$

we see that the two equations of motion are the same if we take

$$\frac{Mg h_i}{I_i} = g / l_i$$

It is convenient to define the radius of gyration of a compound pendulum such that if all its mass $M$ were at a distance from $O_i$ the moment of inertia about $O_i$ would be $I_i$, which we do by writing

$$I_i = M k_i^2$$

Inserting this definition into equation (1) shows that

$$k_i^2 = h_i$$

If $I_G$ is the moment of inertia of the pendulum about its centre of mass $G$, we can also define the radius of gyration about the centre of mass by writing

$$I_G = M k_G^2$$

The parallel axis theorem gives us

$$k_i^2 = k_i^2 + k_G^2$$

so that, using (2), we have

$$k_i = \frac{h_i^2 + k_G^2}{h_i}$$

The period of the pendulum from either suspension point is then

$$T_i = 2\pi \sqrt{\frac{I_i}{g h_i}} = 2\pi \sqrt{\frac{h_i^2 + k_G^2}{g h_i}}$$

Squaring (3), one can show that

$$h_1^2 k_1^2 - h_2^2 k_2^2 = \frac{4 \pi^2}{g} \left( h_1^2 - h_2^2 \right)$$
\[
\frac{4\pi^2}{g} = \frac{\beta_1 \beta_2 - \beta_2 \beta_1}{(\beta_1 + \beta_2)(\beta_1 - \beta_2)} = \frac{T_1^2 + T_2^2}{2(\beta_1 + \beta_2)} + \frac{T_1^2 - T_2^2}{2(\beta_1 - \beta_2)}
\]

which allows us to calculate \( g \),

\[
g = 8\pi^2 \left[ \frac{T_1^2 + T_2^2}{\beta_1 + \beta_2} + \frac{T_1^2 - T_2^2}{\beta_1 - \beta_2} \right]^{-1}
\]

\( (4) \)

**Applications**

Pendulums are used to regulate pendulum clocks, and are used in scientific instruments such as accelerometers and seismometers. Historically they were used as gravimeters to measure the acceleration of gravity in geophysical surveys, and even as a standard of length. The problem with using pendulums proved to be in measuring their length.

A fine wire suspending a metal sphere approximates a simple pendulum, but the wire changes length, due to the varying tension needed to support the swinging pendulum. In addition, small amounts of angular momentum tend to creep in, and the centre of mass of the sphere can be hard to locate unless the sphere has highly uniform density. With a compound pendulum, there is a point called the centre of oscillation, a distance \( l \) from the suspension point along a line through the centre of mass, where \( l \) is the length of a simple pendulum with the same period. When suspended from the centre of oscillation, the compound pendulum will have the same period as when suspended from the original suspension point. The centre of oscillation can be located by suspending from various points and measuring the periods, but it is difficult to get an exact match in the period, so again there is uncertainty in the value of \( l \).

With equation (4), derived by Friedrich Bessel in 1826, the situation is improved. \( h_1 + h_2 \), being the distance between the knife edges, can be measured accurately. \( h_1 - h_2 \) is more difficult to measure accurately, because accurate location of the centre of mass \( G \) is difficult. However, if \( T_1 \) and \( T_2 \) are very nearly equal, the second term in (4) is quite small compared to the first, and \( h_1 - h_2 \) does not have to be known as accurately as \( h_1 + h_2 \).

Kater’s pendulum was used as a gravimeter to measure the local acceleration of gravity with greater accuracy than an ordinary pendulum, because it avoids having to measure \( l \). It was popular from its invention in 1817 until the 1950’s, when began to be possible to directly measure the acceleration of gravity during free fall using a Michelson interferometer. Such an absolute gravimeter is not particularly portable, but it can be used to accurately calibrate a relative gravimeter consisting of a mass hanging from a spring adjacent to an accurate length scale. The relative gravimeter can then be carried to any location where it is desired to measure the acceleration of gravity.
Kater's pendulum

Procedure

Real Lab

- Shift the weight $W_1$ to one end of katers pendulum and fix it. Fix the knife edge $K_1$ just below it.
- Keep the knife edge $K_2$ at the other end and fix the wooden weight $W_2$ symmetrical to other end. Keep the small weight 'w' near to centre.
- Suspend the pendulum about the knife edge 1 and take the time for about 10 oscillations. Note down the time $t_1$ using a stopwatch and calculate its time period using equation $T_1 = t_1/10$.
- Now suspend about knife edge $K_2$ by inverting the pendulum and note the time $t_2$ for 10 oscillations. Calculate $T_2$ also.
- If $T_2 \neq T_1$, adjust the position of knife edge $K_2$ so that $T_2 \approx T_1$.
- Balance the pendulum on a sharp wedge and mark the position of its centre of gravity. Measure the distance of the knife-edge $K_1$ as $h_1$ and that of $K_2$ as $h_2$ from the centre of gravity.

Observations and Calculations

To determine $T_1$ and $T_2$

<table>
<thead>
<tr>
<th>Knife edge</th>
<th>Time for 10 oscillations</th>
<th>Time period</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1(s)</td>
<td>2(s)</td>
</tr>
<tr>
<td>$K_1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$K_2$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Distance of $K_1$ from C.G, $h_1$ = ............m.
Distance of $K_2$ from C.G, $h_2$ = ............m.

$$g = \frac{3\pi^2}{\frac{T_1^2 + T_2^2}{h_1 + h_2} + \frac{T_1^2 - T_2^2}{h_1 - h_2}}$$

Acceleration due to gravity, $g$ = ............ms$^{-2}$.

Simulation

- Choose desired environment from combo box.
- Select suitable values for mass of wood cylinder and mass of steel cylinder.
- Choose the position of knife edge, steel and wood cylinder by changing the sliders for it.
- After choosing values, place the mouse over wood cylinder, drag and make it to oscillate.
- Note the time for 10 oscillations, by clicking on the 'START' and 'STOP' button of stop watch.
- Click on 'STOP' button under variables, for the pendulum to stop oscillating.
- Click on the 'Invert' button to invert the pendulum.
- Again take the time for 10 oscillations.
- Click on 'Show Measurements' checkbox to get the value of $h_1$ and $h_2$.
- Repeat the experiment by changing the values in 'variables' region.

Note - When you click on the wood or steel cylinder after selecting all the variables, a box appears if the selected values is not applicable.

Result

The acceleration due to gravity at a given place is found to be = ............ms$^{-2}$.
7. Newton's Rings-Wavelength of light
Aim:

1. To revise the concept of interference of light waves in general and thin-film interference in particular.
2. To set up and observe Newton’s rings.
3. To determine the wavelength of the given source.

Thin film interference:

A film is said to be thin when its thickness is about the order of one wavelength of visible light which is taken to be 550 nm. When light is incident on such a film, a small portion gets reflected from the upper surface and a major portion is transmitted into the film. Again a small part of the transmitted component is reflected back into the film by the lower surface and the rest of it emerges out of the film. These reflected beams reunite to produce interference. Also the transmitted beams too interfere. This type of interference that takes place in thin films is called interference by division of amplitude.

\[ \theta_1 \] angle of incidence at medium 1 to medium 2 boundary.
\[ \theta_2 \] angle of refraction at medium 1 to medium 2 boundary.
\[ \theta_3 \] angle of refraction at medium 2 to medium 3 boundary.
\[ r_{12} \] reflected light from medium 1 to medium 2 boundary.
\[ r_{23} \] reflected light from medium 2 to medium 1 boundary.
\[ r_{21} \] reflected light from medium 2 to medium 3 boundary.
\[ t_{21} \] transmitted light from medium 2 to medium 1 boundary.
\[ t_{23} \] transmitted light from medium 2 to medium 3 boundary.
\[ d \] thickness of the film.

In the above figure the rays \( r_{12} \) and \( t_{21} \) interfere and results in a constructive or destructive interference depending on their path differences, given as,

\[ 2\mu_2 d \cos r_{12} = (2m + 1) \frac{\lambda}{2} \quad \text{constructive interference} \]
\[ 2\mu_2 d \cos r_{12} = m\lambda \quad \text{destructive interference} \]

Where, \( \mu_2 \rightarrow \) refractive index of the medium 2 and \( m = 0, 1, 2, \ldots \rightarrow \) the order of interference.

The transmitted light from \( t_{23} \) can also interfere and result in constructive or destructive interference.

Thin film interference with films of varying thickness (Newton’s rings):

Rings are fringes of equal thickness. They are observed when light is reflected from a plano-convex lens of a long focal length placed in contact with a plane glass plate. A thin air film is formed between the plate and the lens. The thickness of the air film varies from zero at the point of contact to some value \( t \). If the lens plate system is illuminated with monochromatic light falling on it normally, concentric bright and dark interference rings are observed in reflected light. These circular fringes were discovered by Newton and are called Newton’s rings.

A ray AB incident normally on the system gets partially reflected at the bottom curved surface of the lens (Ray 1) and part of the transmitted ray is partially reflected (Ray 2) from the top surface of the plane glass plate. The rays 1 and 2 are derived from the same incident ray by division of amplitude and therefore are coherent. Ray 2 undergoes a phase change of \( \pi \) upon reflection since it is reflected from air-to-glass boundary.
The condition for constructive and destructive interferences are given as:
for normal incidence $\cos r = 1$ and for air film $\mu = 1$.

$$2t = (2m + 1)\frac{\lambda}{2} \quad \text{constructive interference}$$

$$2t = m\lambda \quad \text{destructive interference}$$

1. **Central dark spot**: At the point of contact of the lens with the glass plate the thickness of the air film is very small compared to the wavelength of light therefore the path difference introduced between the interfering waves is zero. Consequently, the interfering waves at the centre are opposite in phase and interfere destructively. Thus a dark spot is produced.

2. **Circular fringes with equal thickness**: Each maximum or minimum is a locus of constant film thickness. Since the locus of points having the same thickness fall on a circle having its centre at the point of contact, the fringes are circular.

3. **Fringes are localized**: Though the system is illuminated with a parallel beam of light, the reflected rays are not parallel. They interfere nearer to the top surface of the air film and appear to diverge from there when viewed from the top. The fringes are seen near the upper surface of the film and hence are said to be localized in the film.

4. **Radii of the $m$th dark rings**: $r_m = \sqrt{m\lambda R} \cdot$

5. **Radii of the $m$th bright ring**: $r_m = \sqrt{(2m + 1)\lambda R / 2} \cdot$

6. The radius of a dark ring is proportional to the radius of curvature of the lens by the relation, $r_m \propto \sqrt{R}$.

7. Rings get closer as the order increases ($m$ increases) since the diameter does not increase in the same proportion.

8. In transmitted light the ring system is exactly complementary to the reflected ring system so that the centre spot is bright.

9. Under white light we get coloured fringes.

10. The wavelength of monochromatic light can be determined as, $\lambda = \frac{D_{m+p}^2 - D_m^2}{4 \, p \, R} \cdot$

    Where, $D_{m+p}$ is the diameter of the $(m+p)^{th}$ dark ring and $D_m$ is the diameter of the $m^{th}$ dark ring.
Performing Real Lab

After experimental arrangement, the glass plate is inclined at an angle 45° to the horizontal. This glass plate reflects light from the source vertically downloads and falls normally on the convex lens. Newton’s rings are seen using a long focus microscope, focussed on the air film. The cross-wire of the microscope is made tangential to the 20th ring on the left side of the centre. The readings of the main scale and vernier scale of the microscope are noted. The cross wire is adjusted to be tangential to the 18th, 16th, 14th, etc on the left and 2nd, 4th, 6th, etc on the right and readings are taken each time. From this the diameter of the ring is found out which is the difference between the readings on the left and right sides. The square of the diameter and hence \( D_n^2 \) and \( D_{n+m}^2 \) are found out. Then wavelength is calculated using equation.

Performing the simulation:

The simulation virtualizes the Newton’s rings experiment. The user can view the effect of Newton’s rings formed when the medium changes. Select any one type of medium. Different ring pattern can be seen by changing the radius of curvature of the lens and wavelength of light source.

Components:

Start button, Light source, Filter, Microscope, Lens, Medium and Glass plate.

Help:

Variable region:
1. Choose Medium Combo box helps you to choose the type of medium that the simulation have to perform.
2. Radius Slider helps to change the radius of curvature of lens.
3. The wavelength slider helps to change the wavelength of light used.

Measurement region:
1. The start button will help to play the simulation.
2. The variation in the rings can be seen when the medium, wavelength of light or the radius of the lens changes.

Procedure:
1. Click on the "light on" button.
2. Select the lens of desirable radius.
3. Adjust the microscope position to view the Newton rings.
4. Focus the microscope to view the rings clearly.
5. Fix the cross-wire on 20th ring either from right or left of the centre dark ring and take the readings.
6. Move the crosswire and take the reading of 18th,16th............2nd ring.
7. You have to take the reading of rings on either side of the centre dark ring.
8. Enter the readings in the tabular column.
9. Calculate the wavelength of the source by using the given formula.

Observations:

To find Least Count
One main scale division = ............... cm
Number of divisions on Vernier = ............... \( L.C = \frac{\text{One main scale division}}{\text{Number of division on vernier}} \)

<table>
<thead>
<tr>
<th>Order of ring</th>
<th>Microscopic Reading (cm)</th>
<th>Diameter (cm)</th>
<th>( D^2 ) (cm²)</th>
<th>( D_{m+p}^2 - D_m^2 ) (cm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left</td>
<td>Right</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Calculation:

Mean value of \( D_{m+p}^2 - D_m^2 \) =.......cm²
Wavelength of light \( \lambda = \frac{(D_{m+p}^2 - D_m^2)/4pR}{\text{nm}} \)

Result:

Wavelength of light from the given source is found to be = ........nm
8. Young's Modulus-Uniform Bending
Aim
To find the Young's modulus of the given material bar by uniform bending using pin and microscope method.

Apparatus
Pin and Microscope arrangement, Scale, Vernier calipers, Screw gauge, Weight hanger, Material bar or rod.

Theory
Young's modulus is named after Thomas Young, 19th century, British scientist. In solid mechanics, Young's modulus is defined as the ratio of the longitudinal stress over longitudinal strain, in the range of elasticity the Hook's law holds (stress is directly proportional to strain). It is a measure of stiffness of elastic material.

If a wire of length L and area of cross-section 'a' be stretched by a force F and if a change (increase) of length 'l' is produced, then

\[
\text{Young's modulus} = \frac{\text{Normal stress}}{\text{Longitudinal strain}} = \frac{F/a}{l/L}
\]

Uniform Bending Using Pin and Microscope Method
In uniform Bending, the Young's modulus of the material of the bar is given by

\[
Y = \frac{mgp^2}{8Ie}
\]  (1)

Where,
- m - Mass at each end of the bar.
- p - Distance between the point of suspension of the mass and nearer knife edge.
- g - Acceleration due to gravity.
- l is the length of the bar between the knife edges.
- e - Elevation of the midpoint of the bar for a mass m at each end.
- I - Geometrical moment of inertia.

For a bar of rectangular cross section,

\[
I = \frac{bd^3}{12}
\]  (2)

Where b is the breadth and d is the thickness of the bar.

Substituting (2) in equation (1)

\[
Y = \frac{3mgp^2}{2bd^2 e}
\]  (3)

Applications
1. Thin film applications.
2. It helps to predict the directional and orientation properties of metals and has application in ceramics.
3. Measurement of soft tissues - early detection, elasticity imaging, etc.
4. It is used to test equipments like ultrasonic transducers, ultrasonic sensors.
**Procedure for Simulation**

1. Select the environment and material for doing experiment.
2. Adjust length, breadth and thickness of the material bar using sliders on the right side of the simulator.
3. Fix the distance between knife edges and weight hangers using sliders.
4. Focussing the microscope using focussing knob and adjusting the tip of the pin coincides with the point of intersection of the cross wires using left and top knobs on microscope respectively.
5. Readings are noted using the microscope reading for 0g. Zoomed part of microscope scale is available by clicking the centre part of the apparatus in the simulator. Total reading of microscope is MSR+VSR+LC. MSR is the value of main scale reading of the microscope which is coinciding exactly with the zero of vernier scale. One of the divisions in the vernier scale coincides exactly with the main scale is the value of VSR. LC is the least count.
6. Weights are added one by one say 50g, then pin moves downwards while viewing through microscope. Again adjust the pin such that it coincides exactly with the cross wire.
7. Note the microscope reading and repeat 7 and 8 by increasing the weights.
8. The readings are tabulated and Y is determined using equation (3).

**Procedure for Real lab**

**Uniform Bending**

The bar is placed symmetrically on two knife edges. Two weight hangers are suspended at equal distance from the knife edges. The distance between knife edges and distance of the weight hanger from knife edges are measured. A pin is fixed vertically at the midpoint of the bar with its pointed end upwards. The microscope is arranged in front of the pin and focused at the tip of the pin. The slotted weights are added one by one on both the weight hangers and removed one by one a number of times, so that the bar is brought into an elastic mood. With the some "dead load" \( W_0 \) on each weight hanger, the microscope is adjusted so that the image of the tip of the pin coincides with the point of intersection of cross wires. The reading of the vernier scale and vernier of microscope are taken. Weights are added one by one and corresponding reading are taken. From these readings, the mean elevation \( e \) of the mid-point of the bar for a given mass is determined.

The value of \( \frac{p^2}{e} \) is calculated. The breadth of the bar \( b \) is measured by using vernier calipers and thickness of the bar \( d \) is measured by using screw gauge. Hence calculate the Young's modulus of the material bar.

**Observations and Calculations of Uniform Bending**

<table>
<thead>
<tr>
<th>No</th>
<th>Distance of knife edges, ( l ) (cm)</th>
<th>Distance between weight hanger and knife edges, ( p ) (cm)</th>
<th>Load ( M ) (kg)</th>
<th>Telescope reading</th>
<th>Elevation for load ( 4m, e ) (cm)</th>
<th>Mean elevation ( e ) (cm)</th>
<th>Mean of ( \frac{p^2}{e} ) (cm(^2))</th>
<th>Mean of ( \frac{p^2}{e} ) (cm(^2))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( W_0 )</td>
<td>( W_0+m )</td>
<td>( X_0 )</td>
<td>( X_1 )</td>
<td>( X_2 )</td>
<td>( X_3 )</td>
<td>( X_4 )</td>
<td>( X_4 )</td>
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<tr>
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<td>( X_4 )</td>
<td>( X_5 )</td>
<td>( X_6 )</td>
<td>( X_7 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>( W_0+4m )</td>
<td>( W_0+5m )</td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>( W_0+6m )</td>
<td>( W_0+7m )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| Thickness of the material bar \( "d" \) = | ........................................ mm |
| Breadth of the material bar \( "b" \) = | ........................................ cm |
| Mean value of \( \frac{p^2}{e} \) = | ........................................ m |
Young's modulus of the material bar, $Y = \frac{3mgp^2}{2bd'e} = \text{......................... Nm}^{-2}$

Example: For uniform bending for wood, $p=0.5\text{m}$, $m=0.02\text{kg}$, $g=9.8\text{ms}^{-2}$, $pl^2/e = 2.165\text{ m}^2$, $b=2.956 \times 10^{-2}\text{m}$, $d=50693 \times 10^{-3}\text{m}$.

$Y = 1.1 \times 10^{10}\text{ Nm}^{-2}$

Result

1. Young's modulus of the given material using uniform bending method = .......................... Nm$^{-2}$.