

# INTRODUCTION TO STATIC ANALYSIS



# STATIC ANALYSIS

- Single pile design issues
- Group design issues
- Special design considerations
- Additional design and construction considerations

# STATIC ANALYSIS METHODS

Static analysis methods and computer solutions are used to:

- Calculate pile length for loads
- Determine number of piles
- Determine most cost effective pile type
- Calculate foundation settlement
- Calculate performance under uplift and lateral loads

# STATIC ANALYSIS METHODS

Static analysis methods and computer solutions are an integral part of the design process.

Static analysis methods are necessary to determine the most cost effective pile type.

For a given pile type:

- calculate capacity
- determine pile length
- determine number of piles



Bid Quantity

# STATIC ANALYSIS METHODS

Foundation designer must know design loads and performance requirements.

Many static analysis methods are available.

- methods in manual are relatively simple
- methods provide reasonable agreement with full scale tests
- other more sophisticated methods could be used

Designer should fully know the basis for, limitations of, and applicability of a chosen method.

# BASICS OF STATIC ANALYSIS

Static capacity is the sum of the soil/rock resistances along the pile shaft and at the pile toe.

Static analyses are performed to determine ultimate pile capacity and the pile group response to applied loads.

The ultimate capacity of a pile and pile group is the smaller of the soil rock medium to support the pile loads or the structural capacity of the piles.

# ULTIMATE CAPACITY

$$Q_u = (\text{Design Load} \times \text{FS}) + \text{“other”}$$

“Other” could be the resistance provided by scourable soil

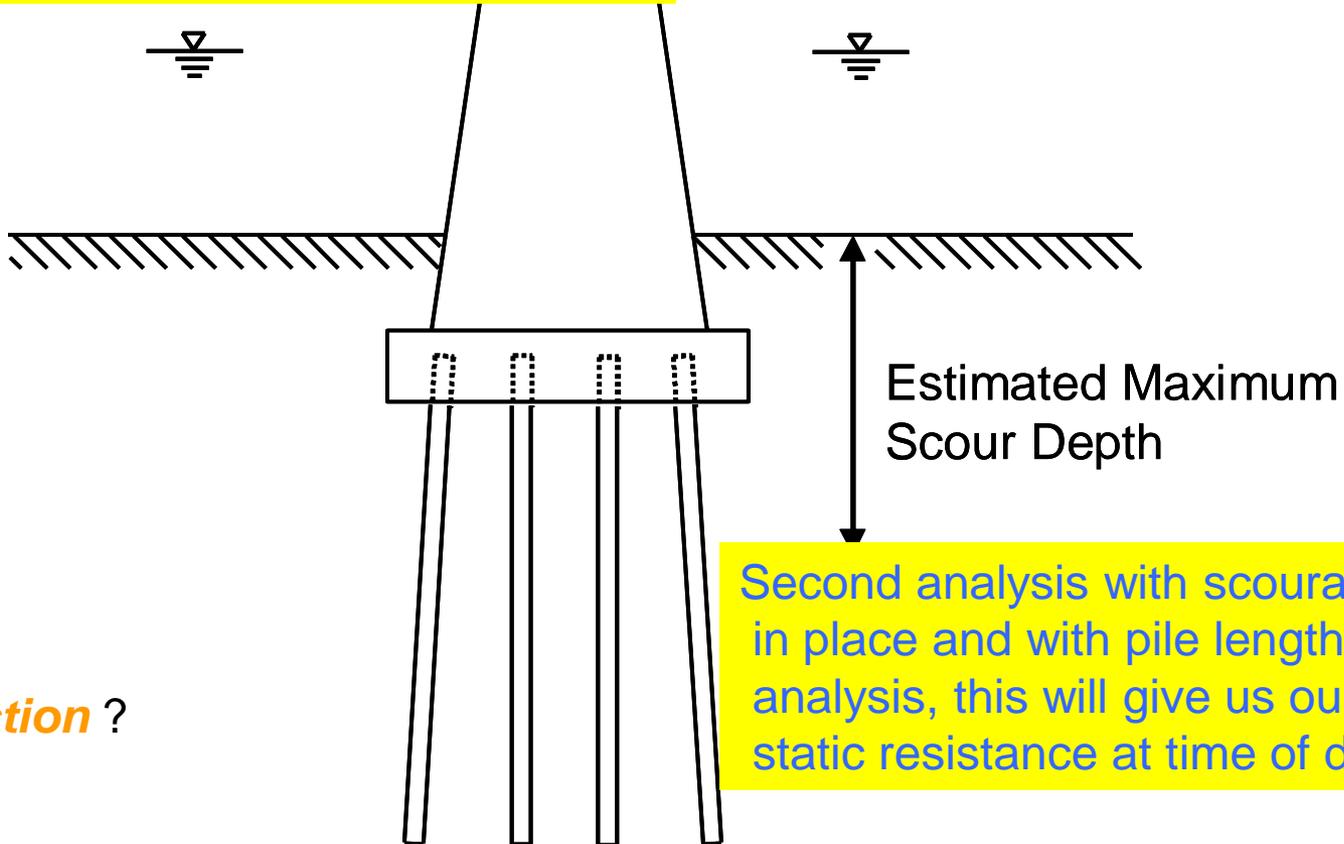
“Other” could be the resistance provided by Liquefiable soil

“Other” is soil resistance at the time of driving not present later during the design life of the pile

# TWO STATIC ANALYSIS ARE OFTEN REQUIRED

First analysis with scourable soil removed, this will give us required pile length for the given ultimate capacity.

Bridge Pier



Estimated Maximum Scour Depth

Second analysis with scourable soil in place and with pile length from first analysis, this will give us our ultimate static resistance at time of driving

*Liquefaction ?*

# LOAD TRANSFER

The ultimate pile capacity is typically expressed as the sum of the shaft and toe resistances:

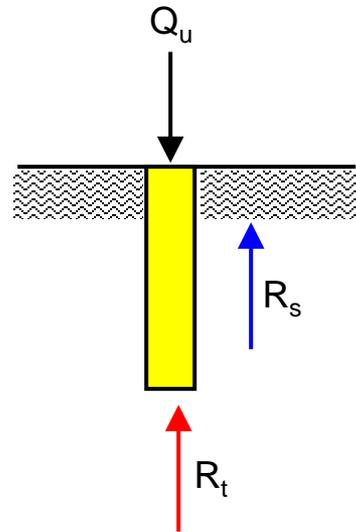
$$Q_u = R_s + R_t$$

This may also be expressed in terms of unit resistances:

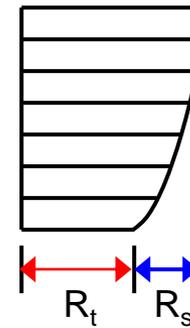
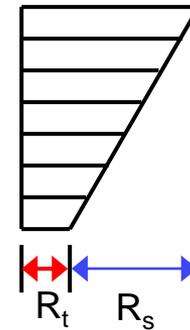
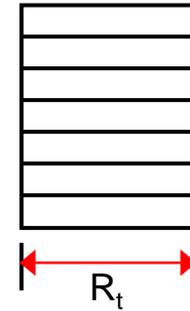
$$Q_u = f_s A_s + q_t A_t$$

The above equations assume that the ultimate shaft and toe resistances are simultaneously developed.

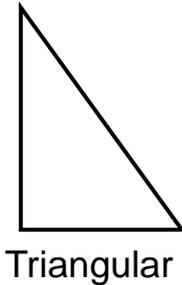
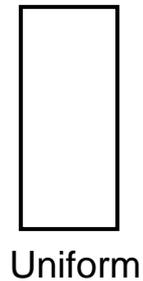
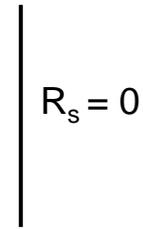
# LOAD TRANSFER



Axial Load vs Depth



Soil Resistance vs Depth



# STUDENT EXERCISE #1

Figure 9.7 on page 12 shows the effect of water table location on effective stresses. Low water table results in higher effective stresses, higher shear strength, and therefore higher driving resistances

# DESIGN SOIL STRENGTH PARAMETERS

Most of the static analysis methods in cohesionless soils use the soil friction angle determined from laboratory tests or SPT N values.

In coarse granular deposits, the soil friction angle should be chosen conservatively.

*What does this mean ??* 

# DESIGN SOIL STRENGTH PARAMETERS

For a cost effective design with any static analysis method, the foundation designer must consider time dependent soil strength changes.

# FACTOR OF SAFETY SELECTION

Historically, the range in factor of safety has depended upon the reliability of a particular static analysis method with consideration of :

- Level of confidence in the input parameters
- Variability of soil and rock
- Method of static analysis
- Effects of, and consistency of proposed pile installation method
- Level of construction monitoring

# FACTORS OF SAFETY

The factor of safety used in a static analysis should be based on the construction control method specified.

| Construction Control Method                  | Factor of Safety |
|--|------------------|
| Static load test with wave equation analysis | 2.00             |
| Dynamic testing with wave equation analysis  | 2.25             |
| Indicator piles with wave equation analysis  | 2.50             |
| Wave equation analysis                       | 2.75             |
| Gates dynamic formula                        | 3.50             |

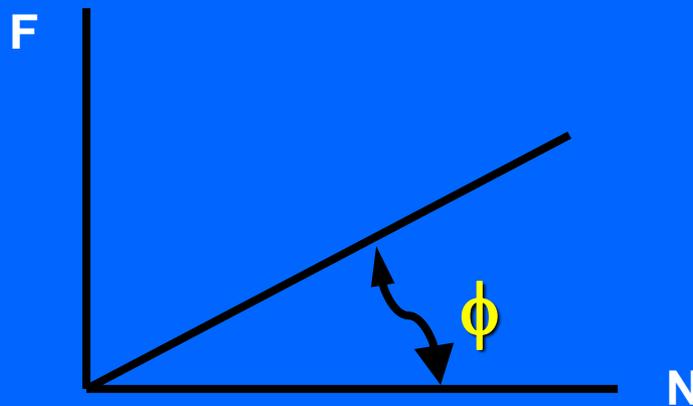
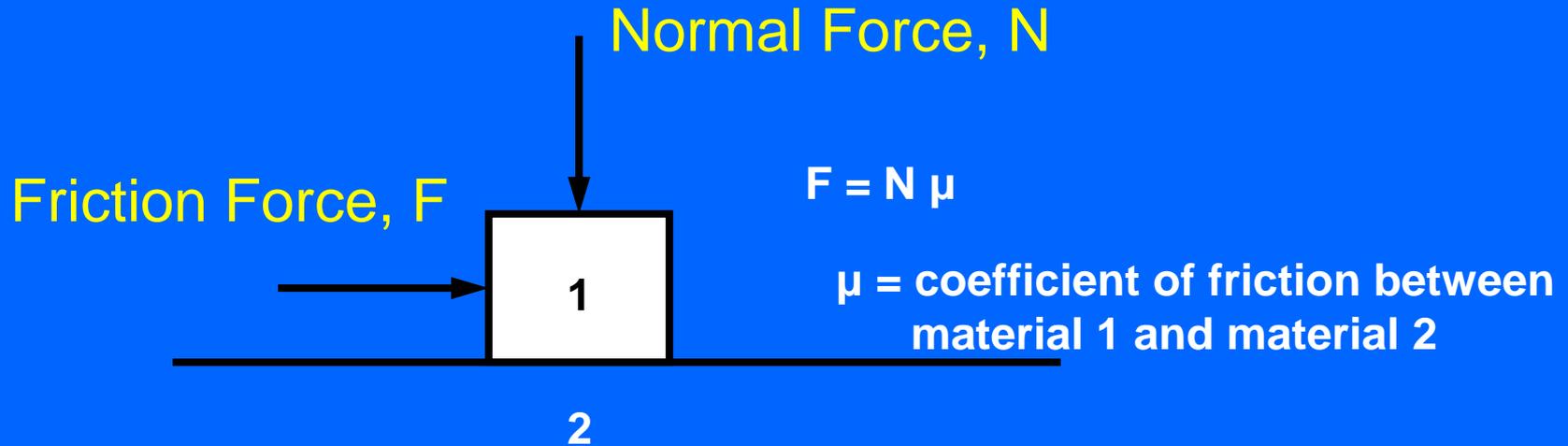
# Static Analysis - Single Piles

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**Methods for estimating static  
resistance of soils**

**STATIC CAPACITY  
OF PILES IN  
COHESIONLESS SOILS**

# Cohesionless Soils, Drained Strength



$$\text{Tan } (\phi) = F/N$$

$$F = N \text{TAN } (\phi)$$

Soil on Soil, we use  $\phi$

Soil on Pile, we use  $\delta$

phi = angle such that TAN ( $\phi$ ) is coefficient of friction between material 1 and material 2

## METHODS OF STATIC ANALYSIS FOR PILES IN COHESIONLESS SOILS

| Method                       | Approach   | Design Parameters  | Advantages  | Disadvantages   | Remarks   |
|------------------------------|--|--|---|---|---|
| Meyerhof Method              | Empirical<br><br>Experience                          | Results of SPT tests.<br><br>N   | Widespread use of SPT test and input data availability. Simple method to use.   | Non reproducibility of N values. Not as reliable as the other methods presented in this chapter.                          | Due to non reproducibility of N values and simplifying assumptions, use should be limited to preliminary estimating purposes. |
| Brown Method                 | Empirical  | Results of SPT tests based on $N_{60}$ values.                                 | Widespread use of SPT test and input data availability. Simple method to use.   | $N_{60}$ values not always available.   | Simple method based on correlations with 71 static load test results. Details provided in Section 9.7.1.1b.                   |
| Nordlund Method.<br><br>FHWA | Semi-empirical<br><br>Part Theory<br>Part Experience | Charts provided by Nordlund. Estimate of <u>soil friction angle</u> is needed. | Allows for increased shaft resistance of tapered piles and includes effects of pile-soil friction coefficient for different pile materials. | No limiting value on unit shaft resistance is recommended by Nordlund. Soil friction angle often estimated from SPT data. | Good approach to design that is widely used. Method is based on field observations. Details provided in Section 9.7.1.1c.     |

## METHODS OF STATIC ANALYSIS FOR PILES IN COHESIONLESS SOILS

| Method   | Approach       | Design Parameters   | Advantages   | Disadvantages   | Remarks  |
|--|----------------|---|--|---|--|
| Effective Stress Method.                           | Semi-empirical | Soil classification and estimated friction angle for $\beta$ and $N_t$ selection. | $\beta$ value considers pile-soil friction coefficient for different pile materials. Soil resistance related to effective overburden pressure. | Results effected by range in $\beta$ values and in particular by range in $N_t$ chosen. | Good approach for design. Details provided in Section 9.7.1.3. |
| Methods based on Cone Penetration Test (CPT) data. | Empirical      | Results of CPT tests.   | Testing analogy between CPT and pile. Reliable correlations and reproducible test data.  | Limitations on pushing cone into dense strata.  | Good approach for design. Details provided in Section 9.7.1.7. |

# Nordlund Data Base

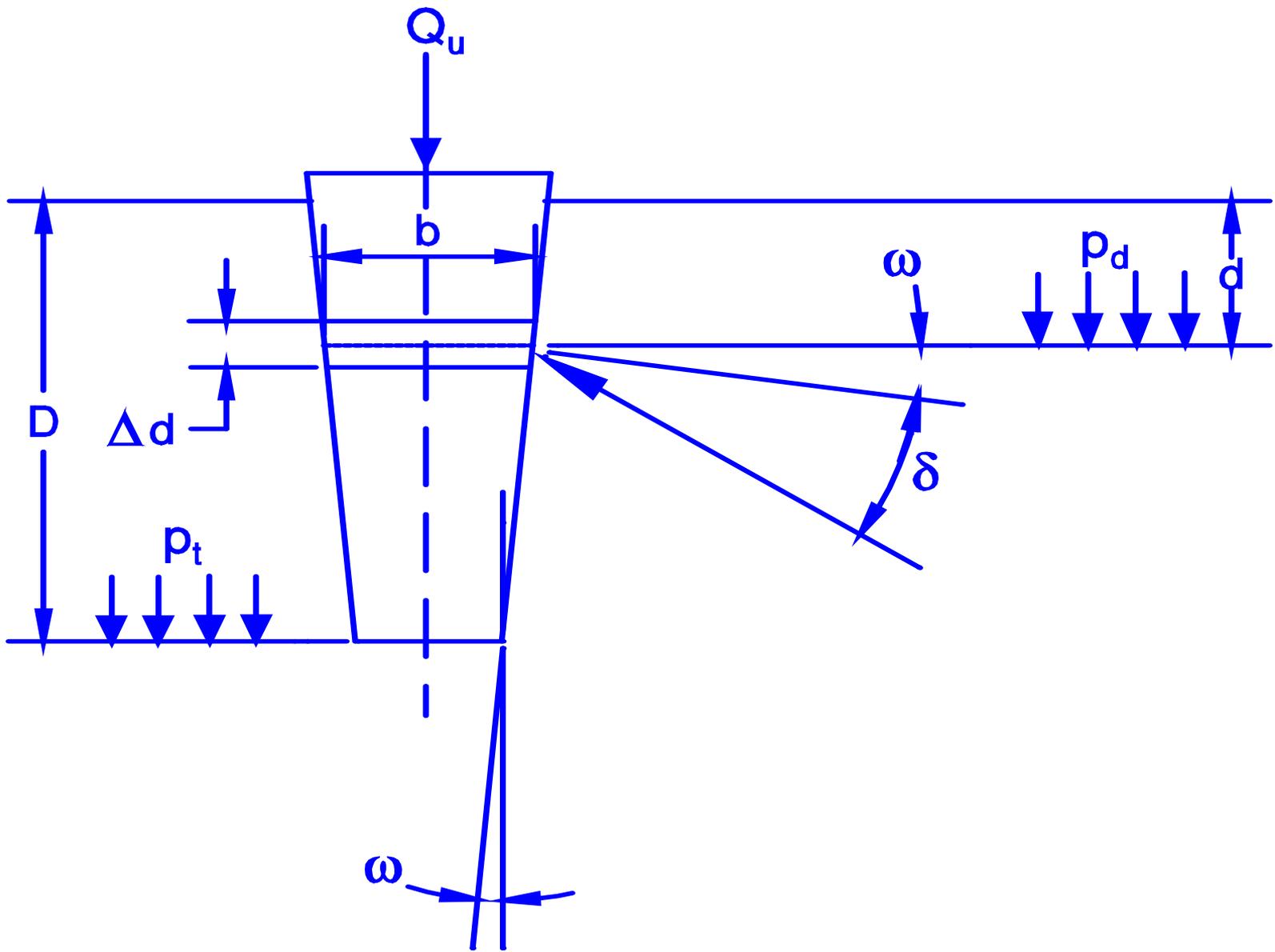
|            |  |
|------------|--|
| Pile Types | Timber, H-piles, Closed-end Pipe, Monotube, Raymond Step-Taper |
| Pile Sizes | Pile widths of 250 – 500 mm (10 - 20 in)                       |
| Pile Loads | Ultimate pile capacities of 350 -2700 kN (40 -300 tons)        |

**Nordlund Method tends to overpredict capacity of piles greater than 600 mm (24 in)**

# Nordlund Method

## Considers:

1. The friction angle of the soil.
2. The friction angle of the sliding surface.
3. The taper of the pile.
4. The effective unit weight of the soil.
5. The pile length.
6. The minimum pile perimeter.
7. The volume of soil displaced.



9-27

$$Q_u = \sum_{d=0}^{d=D} K_{\delta} C_F p_d \frac{\sin(\delta + \omega)}{\cos \omega} C_d \Delta d + \alpha_t N'_q A_t p_t$$

# Nordlund Method

For a pile of uniform cross section ( $\omega=0$ ) and embedded length  $D$ , driven in soil layers of the same effective unit weight and friction angle, the Nordlund equation becomes:

$$Q_u = \underbrace{(K_\delta C_{FP_d}(\sin\delta)C_d D)}_{R_s} + \underbrace{(\alpha_t N'_q A_t p_t)}_{R_T}$$

# Nordlund Shaft Resistance

$$R_s = K_\delta C_F p_d \sin \delta C_d D$$

$K_\delta$  = coefficient of lateral earth pressure      Figures 9.11 - 9.14

$C_F$  = correction factor for  $K_\delta$  when  $\delta \neq \phi$       Figure 9.15

$p_d$  = effective overburden pressure at center of layer

$\delta$  = friction angle between pile and soil      Figure 9.10

$C_d$  = pile perimeter

$D$  = embedded pile length

# Nordlund Toe Resistance

$$R_T = \alpha_T N'_q p_T A_T$$

Lesser of

$$R_T = q_L A_T$$

$\alpha_T$  = dimensionless factor

Figure 9.16a

$N'_q$  = bearing capacity factor

Figure 9.16b

$A_T$  = pile toe area

$p_T$  = effective overburden pressure at pile toe  $\leq 150$  kPa

$q_L$  = limiting unit toe resistance Figure 9.17

# Nordlund Method

$$R_u = R_S + R_T$$

and

$$Q_a = R_U / FS$$

**FS based on construction control method as in 9-14**

# Nordlund Method Procedure

Steps 1 through 6 are for computing shaft resistance and steps 7 through 9 are for computing the pile toe resistance

- STEP 1 Delineate the soil profile into layers and determine the  $\phi$  angle for each layer
- Construct  $p_o$  diagram using procedure described in Section 9.4.
  - Correct SPT field  $N$  values for overburden pressure using Figure 4.4 from Chapter 4 and obtain corrected SPT  $N'$  values. Delineate soil profile into layers based on corrected SPT  $N'$  values.
  - Determine  $\phi$  angle for each layer from laboratory tests or in-situ data.
  - In the absence of laboratory or in-situ test data, determine the average corrected SPT  $N'$  value,  $\bar{N}'$ , for each soil layer and estimate  $\phi$  angle from Table 4-5 in Chapter 4.

# STATIC CAPACITY OF PILES IN COHESIVE SOILS

# Cohesive Soils, Undrained Strength



**F = Friction resistance ; N = Normal force (stress)**

**C is independent of overburden pressures**

**c = cohesion, stickiness, soil / soil**

**a = adhesion, stickiness, soil / pile**

# METHODS OF STATIC ANALYSIS FOR PILES IN COHESIVE SOILS

| Method  | Approach  | Method of Obtaining Design Parameters  | Advantages   | Disadvantages   | Remarks  |
|---|---|--|--|---|--|
| $\alpha$ -Method (Tomlinson Method).<br><br><b style="color: red;">FHWA</b> | Empirical, total stress analysis.                     | Undrained shear strength estimate of soil is needed. Adhesion calculated from Figures 9.18 and 9.19. | Simple calculation from laboratory undrained shear strength values to adhesion.  | Wide scatter in adhesion versus undrained shear strengths in literature.            | Widely used method described in Section 9.7.1.2a.  |
| Effective Stress Method.  | Semi-Empirical, based on effective stress at failure. | $\beta$ and $N_t$ values are selected from Table 9-6 based on drained soil strength estimates.       | Ranges in $\beta$ and $N_t$ values for most cohesive soils are relatively small. | Range in $N_t$ values for hard cohesive soils such as glacial tills can be large.   | Good design approach theoretically better than undrained analysis. Details in Section 9.7.1.3. |
| Methods based on Cone Penetration Test data.                                | Empirical.  | Results of CPT tests.  | Testing analogy between CPT and pile. Reproducible test data.                    | Cone can be difficult to advance in very hard cohesive soils such as glacial tills. | Good approach for design. Details in Section 9.7.1.7.  |

# Tomlinson or $\alpha$ -Method

Unit Shaft Resistance,  $f_s$ :

$$f_s = c_a = \alpha c_u$$

Where:

$c_a$  = adhesion (Figure 9.18)

$\alpha$  = empirical adhesion factor (Figure 9.19)

# Tomlinson or $\alpha$ -Method

Shaft Resistance,  $R_s$ :

$$R_s = f_s A_s$$

Where:

$A_s$  = pile surface area in layer  
(pile perimeter x length)

H piles use “box” area – 4 sides, pg 45

# Tomlinson or $\alpha$ -Method (US)

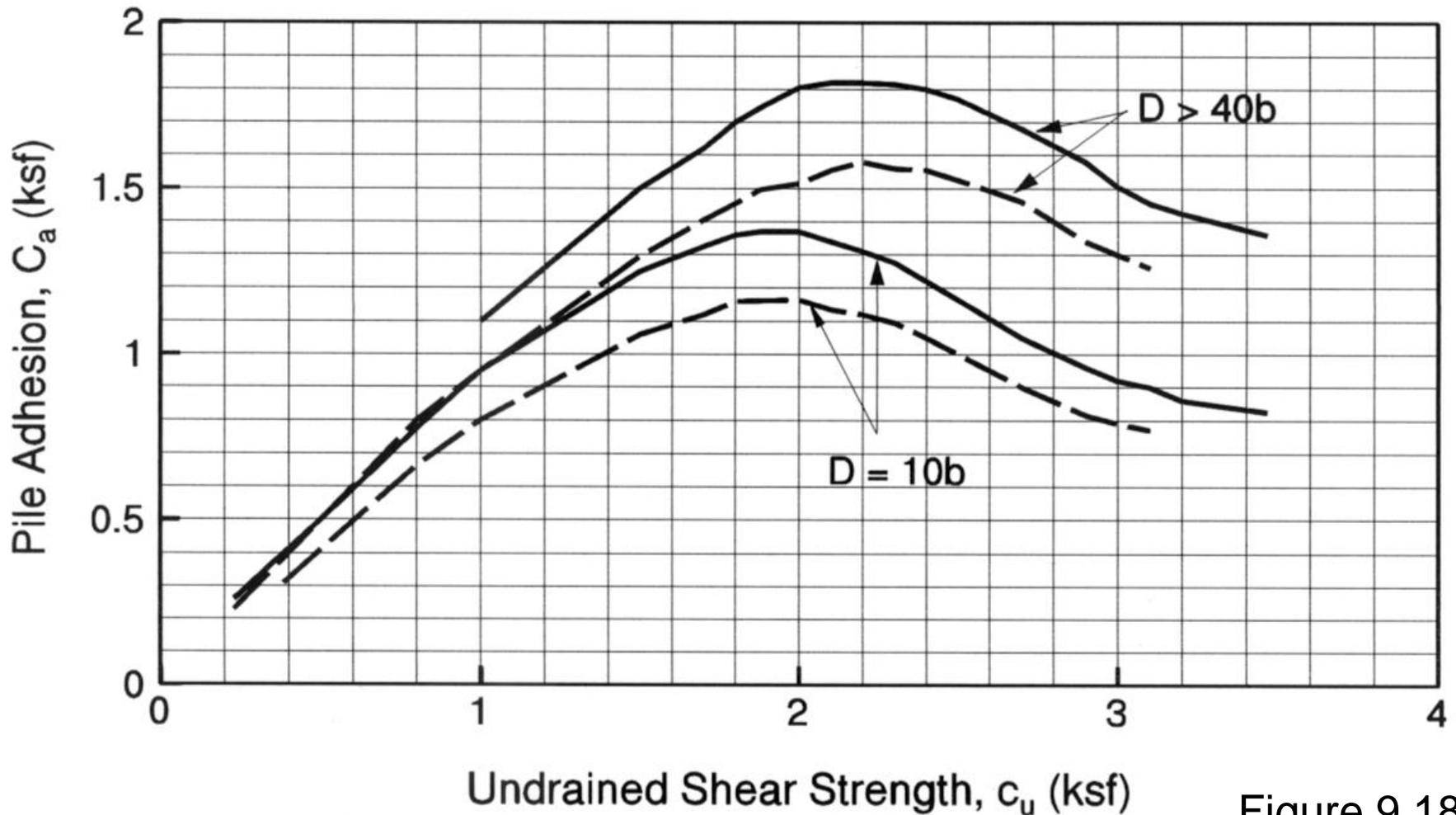
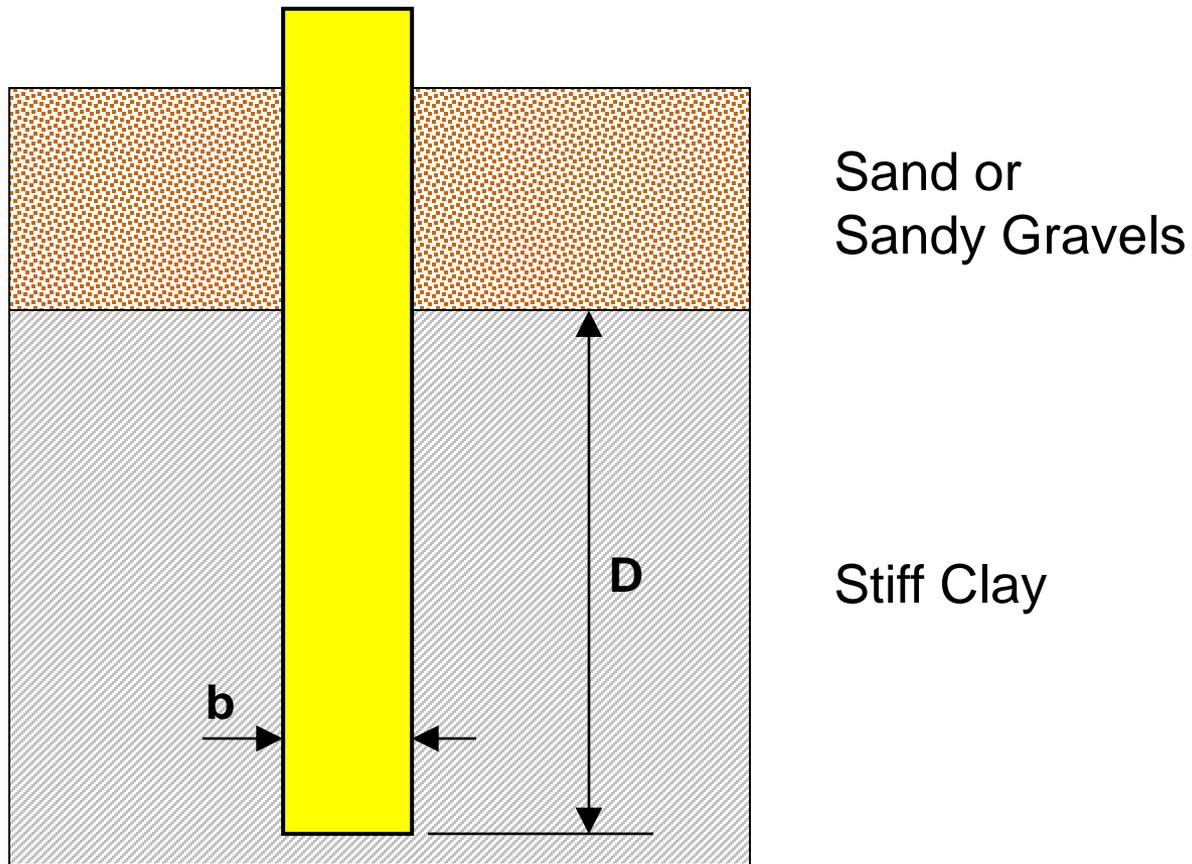


Figure 9.18

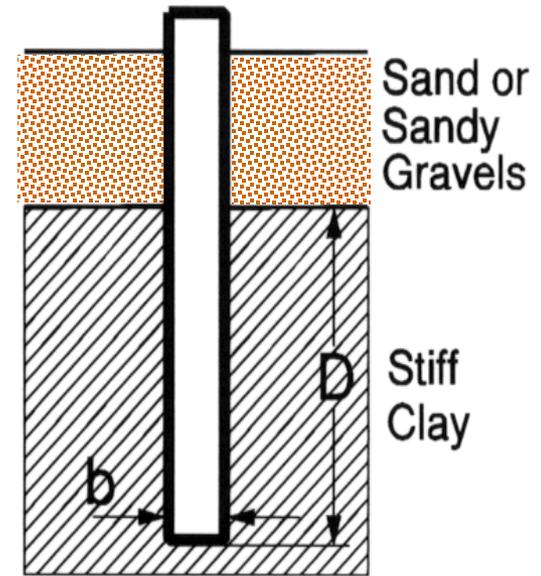
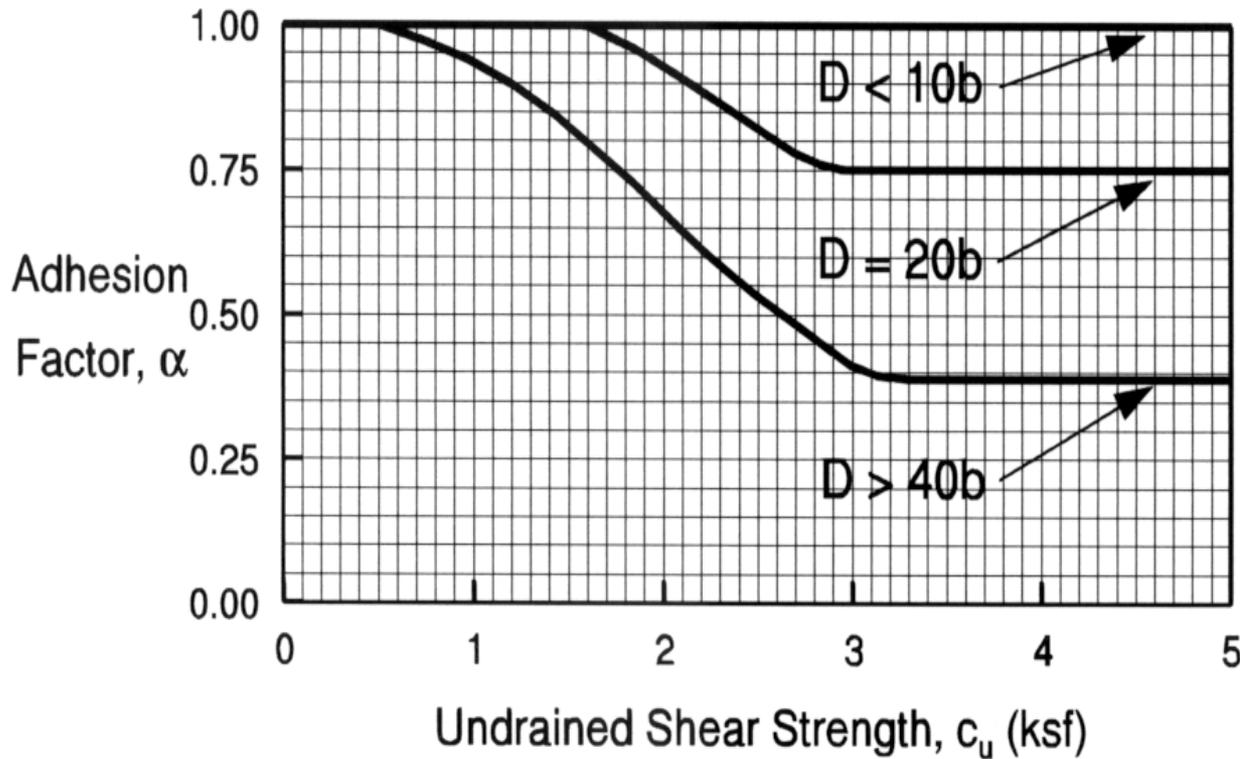
— Concrete, Timber, Corrugated Steel Piles  
- - - Smooth Steel Piles

$D$  = distance from ground surface to bottom of clay layer or pile toe, whichever is less  
 $b$  = Pile Diameter

# Tomlinson or $\alpha$ -Method



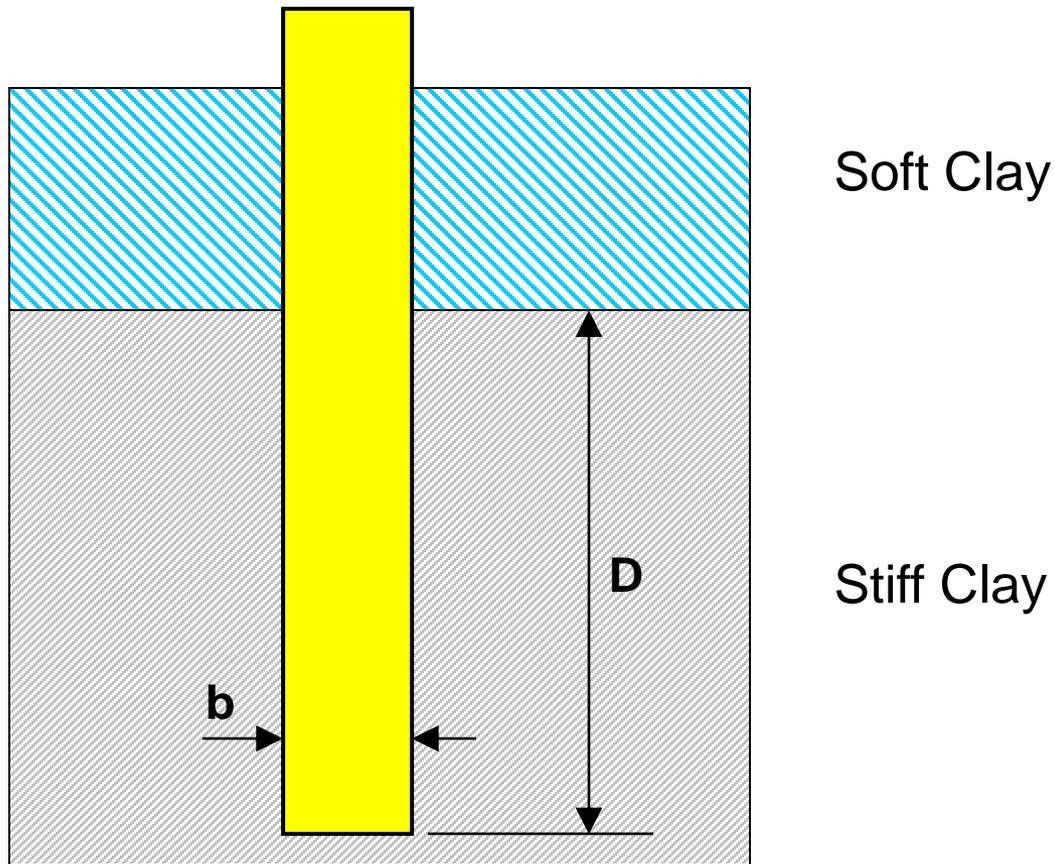
# Tomlinson or $\alpha$ -Method (US)



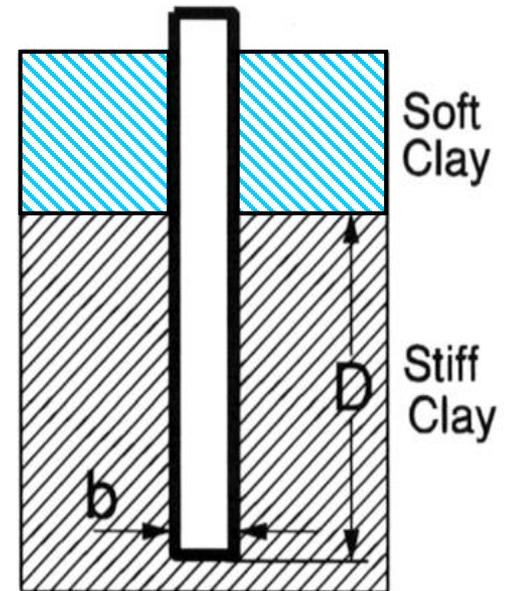
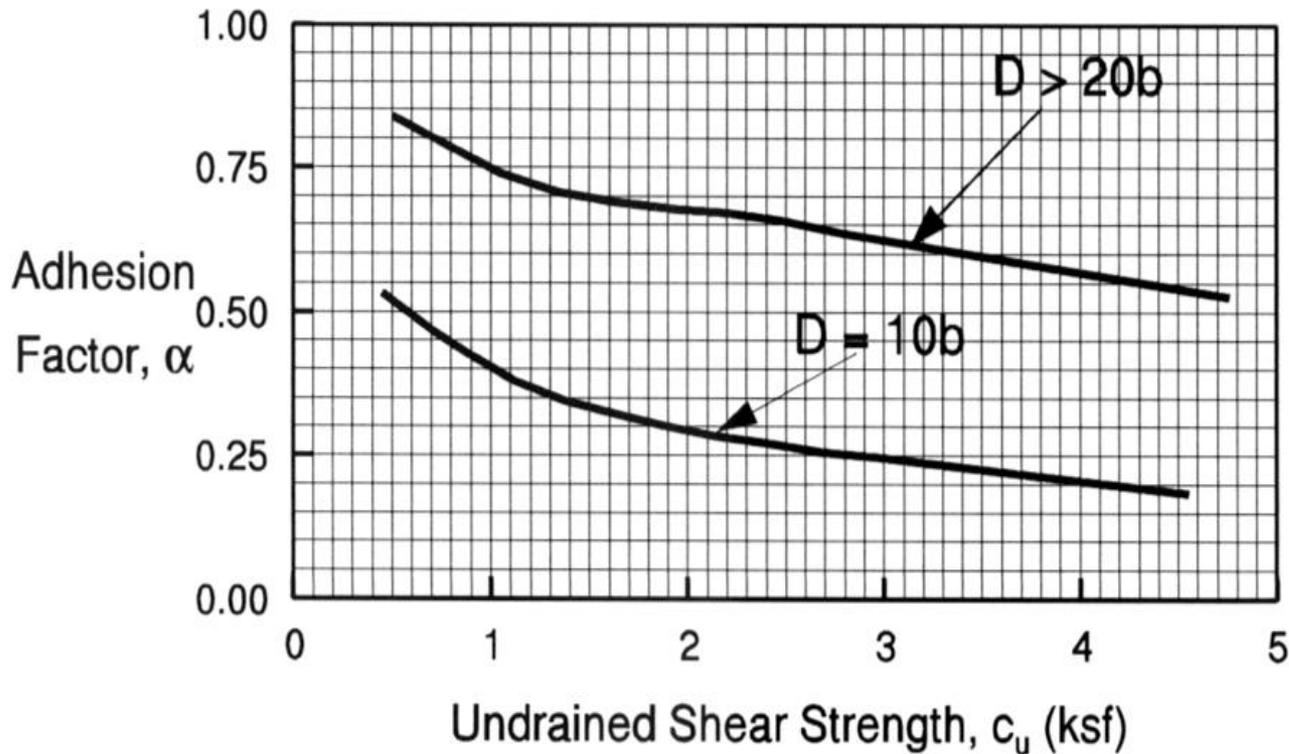
$D$  = distance into stiff clay layer

$b$  = Pile Diameter

# Tomlinson or $\alpha$ -Method



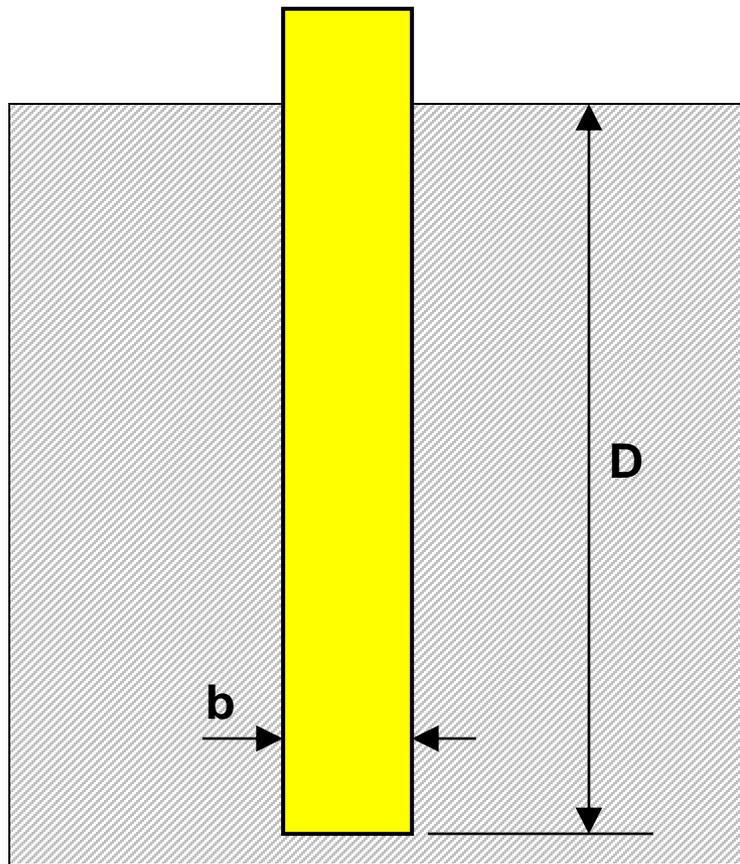
# Tomlinson or $\alpha$ -Method (US)



$D$  = distance into stiff clay layer

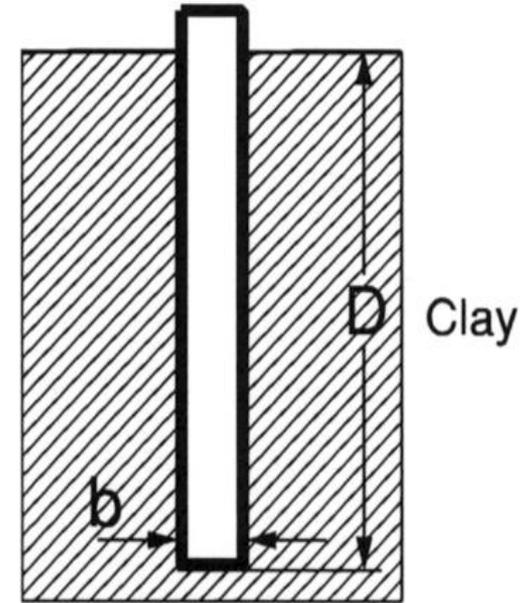
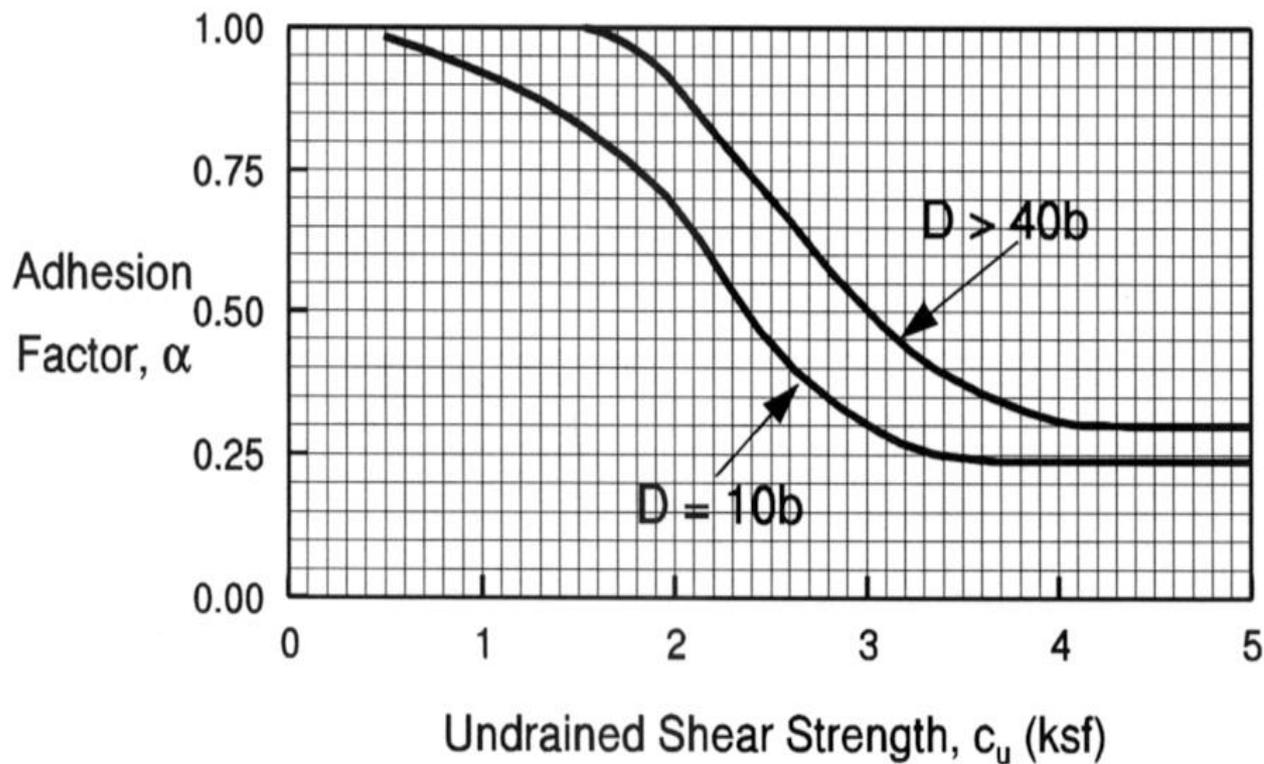
$b$  = Pile Diameter

# Tomlinson or $\alpha$ -Method



Stiff Clay

# Tomlinson or $\alpha$ -Method (US)



$D$  = distance into stiff clay layer

$b$  = Pile Diameter

# Tomlinson or $\alpha$ -Method

Unit Toe Resistance,  $q_t$ :

$$q_t = c_u N_c$$

Where:

$c_u$  = undrained shear strength of the soil at pile toe

$N_c$  = dimensionless bearing capacity factor  
(9 for deep foundations)

# Tomlinson or $\alpha$ -Method

Toe Resistance,  $R_t$ :

$$R_t = q_t A_t$$

The toe resistance in cohesive soils is sometimes ignored since the movement required to mobilize the toe resistance is several times greater than the movement required to mobilize the shaft resistance.

# Tomlinson or $\alpha$ -Method

$$R_u = R_S + R_T$$

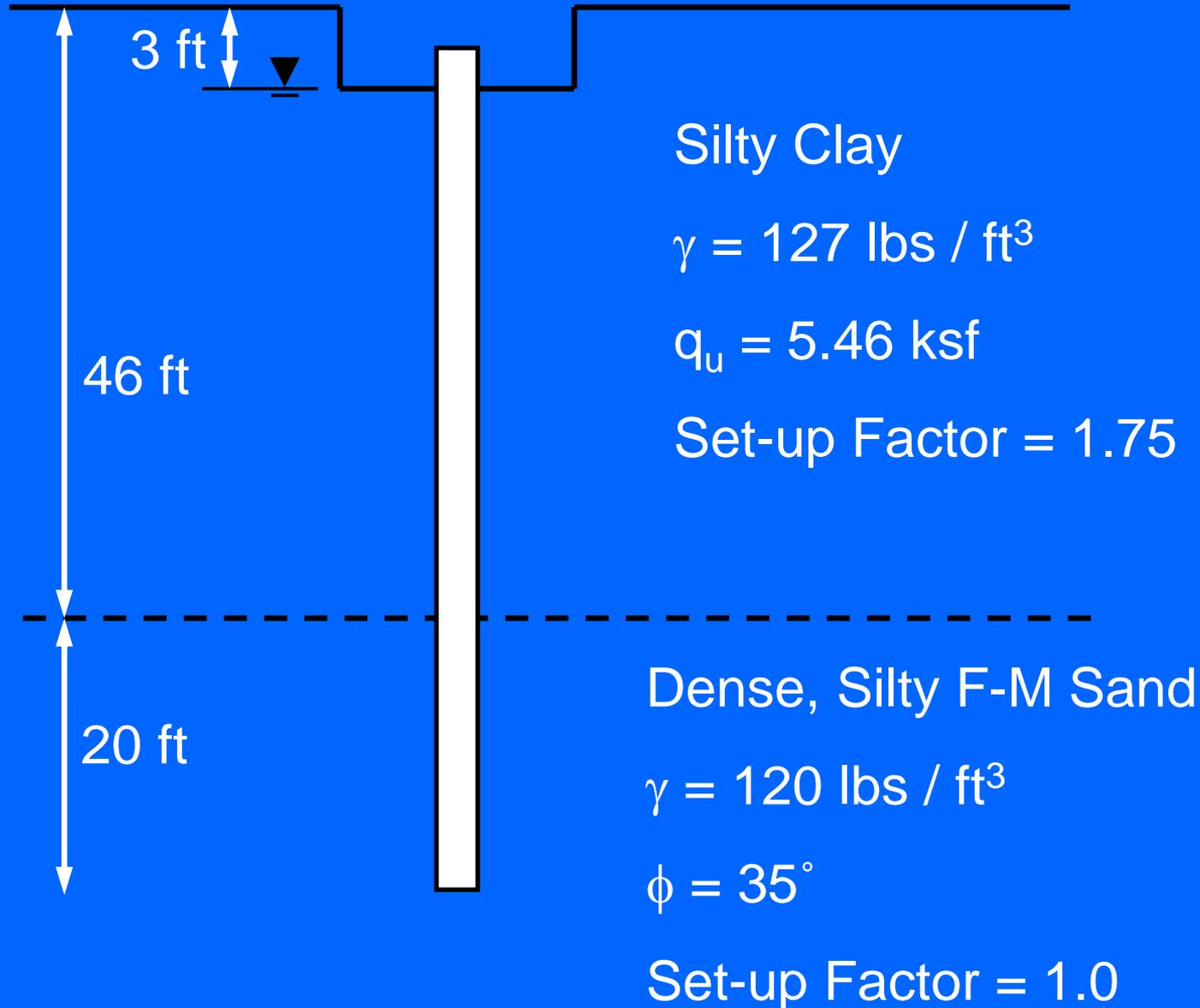
and

$$Q_a = R_U / FS$$

# STUDENT EXERCISE #2

Use the  $\alpha$ -Method described in Section 9.7.1.2a and the Nordlund Method described in Section 9.7.1.1c to calculate the ultimate pile capacity and the allowable design load for a 12.75 inch O.D. closed end pipe pile driven into the soil profile described below. The trial pile length for the calculation is 63 feet below the bottom of pile cap excavation which extends 3 feet below grade. The pipe pile has a pile-soil surface area of 3.38 ft<sup>2</sup>/ft and a pile toe area of 0.89 ft<sup>2</sup>. Use Figure 9.18 to calculate the shaft resistance in the clay layer. The pile volume is 0.89 ft<sup>3</sup>/ft. The effective overburden at 56 feet, the midpoint of the pile shaft in the sand layer is 3.73 ksf, and the effective overburden pressure at the pile toe is 4.31 ksf. Remember, the soil strengths provided are unconfined compression test results ( $c_u = q_u / 2$ ).

# Soil Profile



# Solution

- We will discuss this solution during the DRIVEN workshop (ie steps 1-9)

STEP 10

$$Q_u = R_s + R_t = 1465 + 410 = 1875 \text{ kN}$$

# METHODS BASED ON CPT DATA

Pages 9-65 through 9-80

Nottingham and Schmertmann

Laboratoire des Ponts et Chaussees (LPC)

Elsami and Fellenius

# UPLIFT CAPACITY OF SINGLE PILES

pg 9-81

Increasingly important design consideration

Sources of uplift loads include seismic events, vessel impact, debris loading and cofferdam dewatering.

The design uplift load may be taken as  $\frac{1}{3}$  the ultimate shaft resistance from a static analysis.

# STATIC ANALYSIS - SINGLE PILES LATERAL CAPACITY METHODS

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Reference Manual Chapter 9.7.3

# Lateral Capacity of Single Piles

- Potential sources of lateral loads include vehicle acceleration & braking, wind loads, wave loading, debris loading, ice forces, vessel impact, lateral earth pressures, slope movements, and seismic events.
- These loads can be of the same magnitude as axial compression loads.

# Lateral Capacity of Single Piles

- Historically, prescription values were used for lateral capacity of vertical piles, or battered (inclined) piles were added.
- Modern design methods are readily available which allow load-deflection behavior to be rationally evaluated.

# Lateral Capacity of Single Piles

Soil, pile, and load parameters significantly affect lateral capacity.

- Soil Parameters
  - Soil type & strength
  - Horizontal subgrade reaction
- Pile Parameters
  - Pile properties
  - Pile head condition
  - Method of installation
  - Group action
- Lateral Load Parameters
  - Static or Dynamic
  - Eccentricity

# Lateral Capacity of Single Piles

## Design Methods

- Lateral load tests
- Analytical methods
  - Broms' method, 9-86, (long pile, short pile)
  - Reese's COM624P method
  - LPILE program
  - FB-PIER

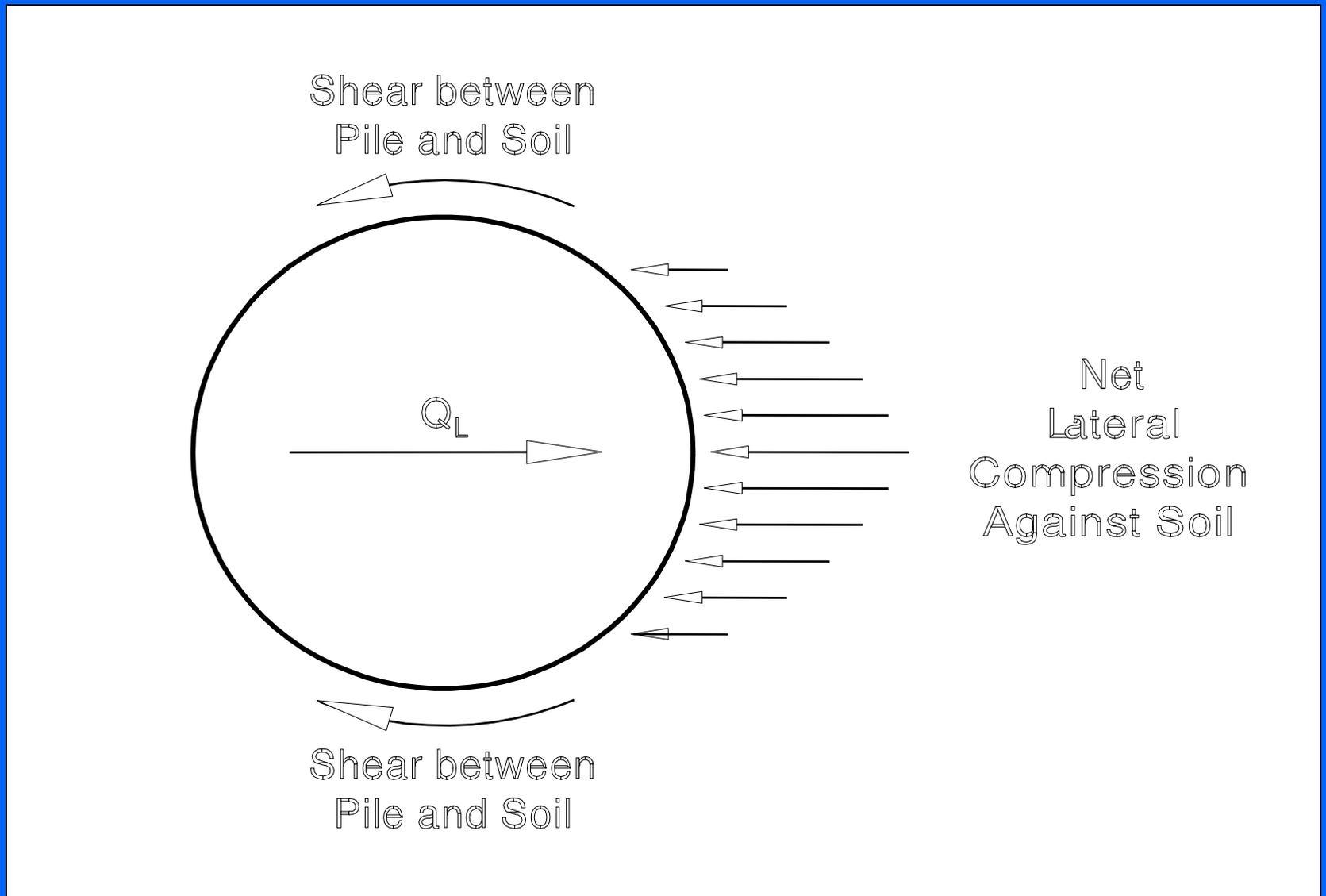


Figure 9.36 Soil Resistance to a Lateral Pile Load (adapted from Smith, 1989)

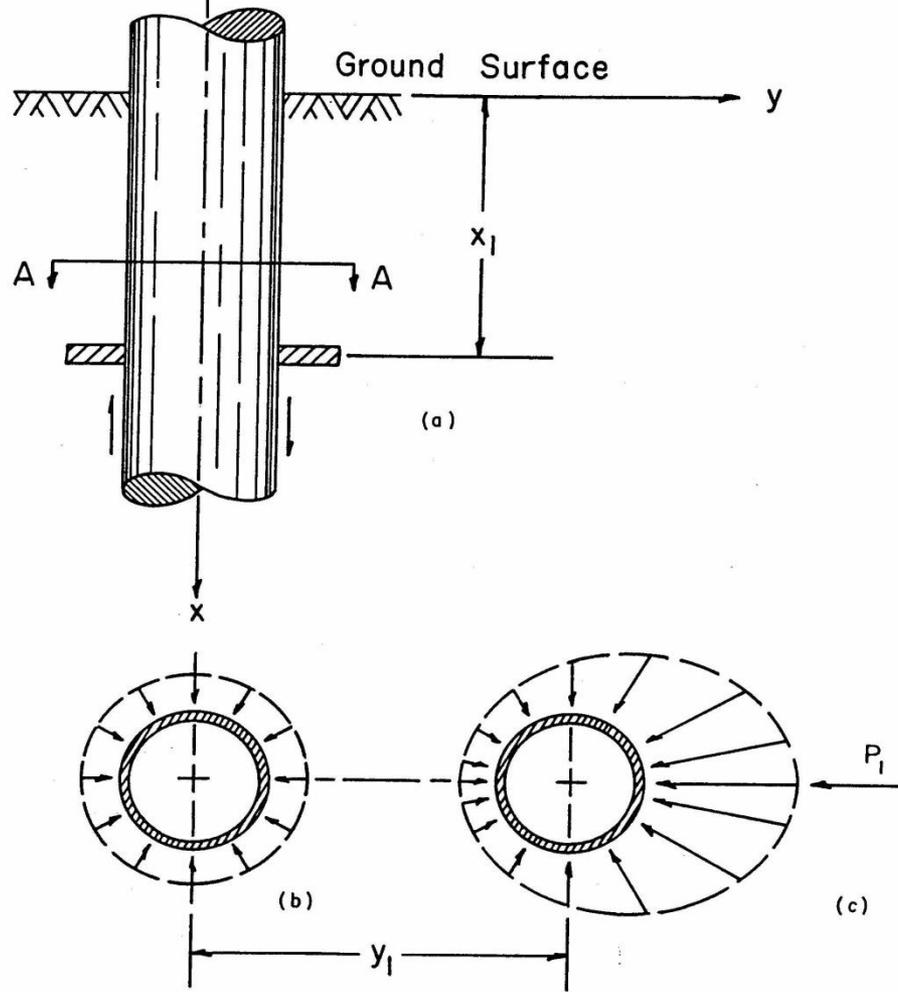


Fig. 3.1. Graphical definition of  $p$  and  $y$   
 (a) view of elevation of section of pile  
 (b) view A-A - earth pressure distribution prior to lateral loading  
 (c) view A-A - earth pressure distribution after lateral loading.

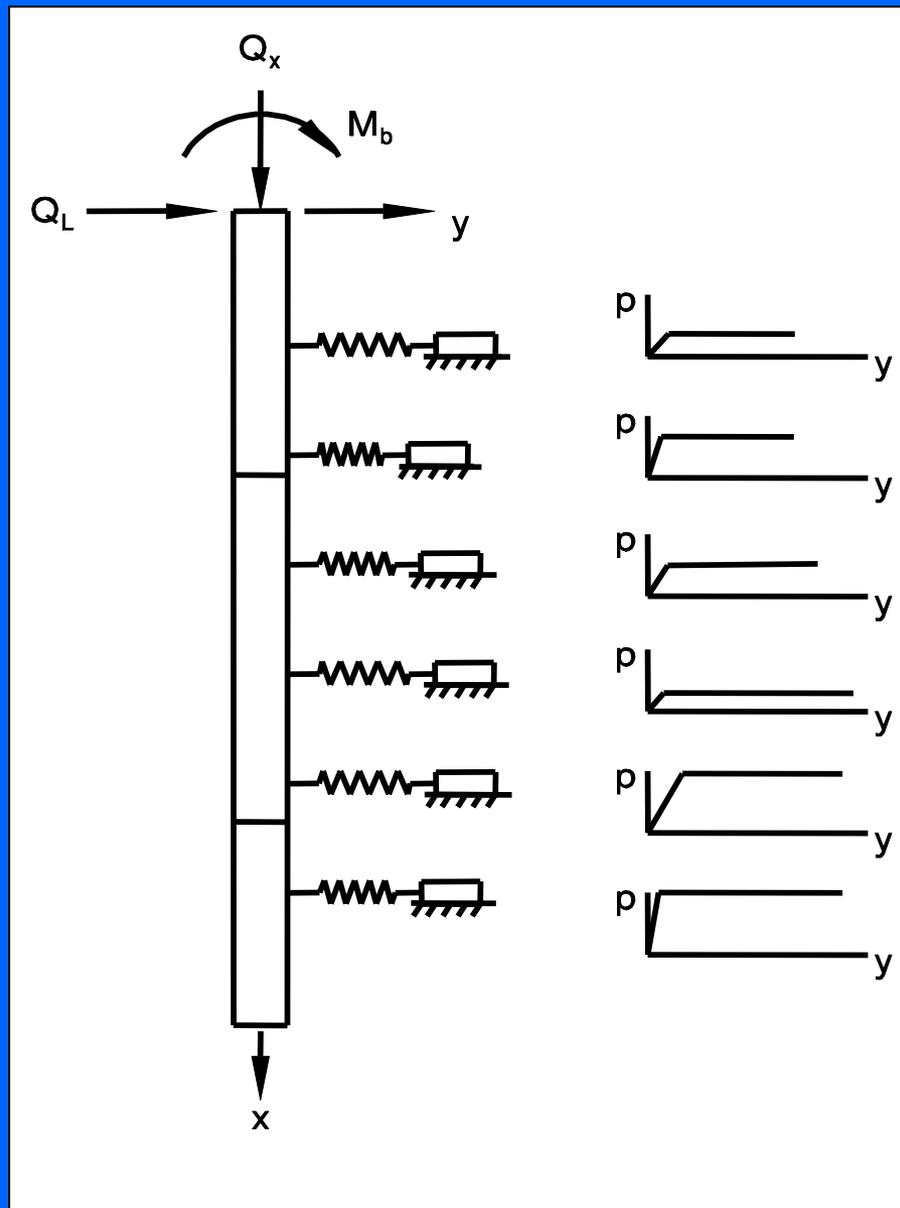
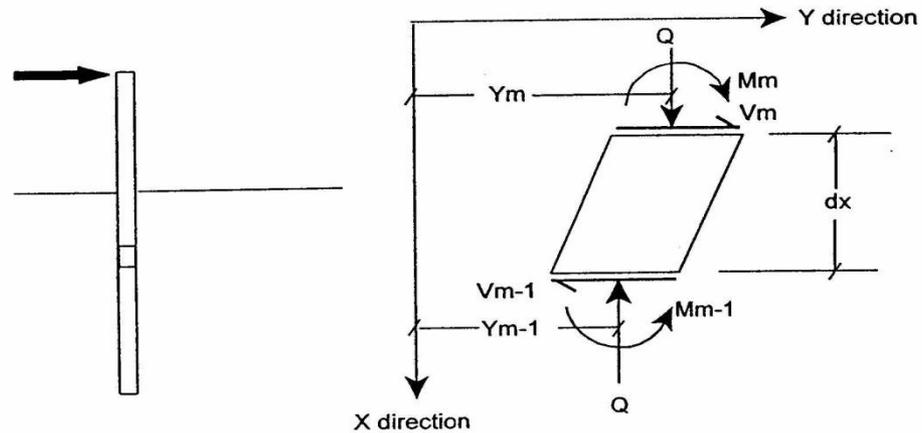


Figure 9.44 LPILE Pile-Soil Model

## Development of general equation for laterally loaded pile



$y_m$  = deflection at top

$y_{m+1}$  = deflection at bottom

$M_m$  = moment at top

$M_{m+1}$  = moment at bottom

$p$  = soil response (force/length)

$Q_m$  = axial load (top and bottom)

$x$  = direction along pile length

$y$  = deflection at some distance,  $x$

Take moments about lower left on  $x$  axis:

$$M_{m-1} - M_m + Q(y_{m-1}) - Q(y_m) - V_m(dx) = 0$$

$$M_{m-1} - M_m - Q(y_m - y_{m-1}) - V_m(dx) = 0$$

$$\Delta M + Q(\Delta y) - Vdx = 0$$

$$dM/dx + Q dy/dx - V = 0$$

differentiate with respect to  $x$ :

$$d^2M/dx^2 + Q d^2y/dx^2 - dV/dx = 0$$

recall that  $dV/dx = p$  (distributed load)  
Recall that  $M = EI d^2y/dx^2$

$$EI d^4y/dx^4 + Q d^2y/dx^2 - p = 0$$

<----- This is the equation that is solved by means of finite differences in most software packages (COM624P, LPILE etc.)

## 5.2 RELATIONSHIPS IN DIFFERENCE FORM

Figure 5.1 shows a portion of the elastic curve of a pile. Relationships in difference form are as follows:

$$\left(\frac{dy}{dx}\right)_{x=m} \cong \frac{y_{m-1} - y_{m+1}}{2h} \quad (5.2)$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=m} \cong \frac{\frac{y_{m-1} - y_m}{h} - \frac{y_m - y_{m+1}}{h}}{h} \cong \frac{y_{m-1} - 2y_m + y_{m+1}}{h^2} \quad (5.3)$$

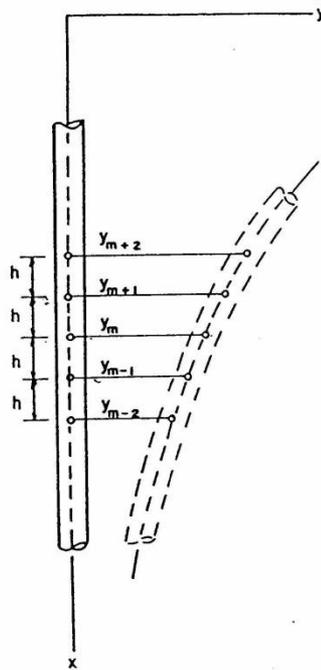


Fig. 5.1. Representation of deflected pile.

In a similar manner

$$\left(\frac{d^3y}{dx^3}\right)_{x=m} \cong \frac{y_{m-2} - 2y_{m-1} + 2y_{m+1} - y_{m+2}}{2h^3} \quad (5.4)$$

$$\left(\frac{d^4y}{dx^4}\right)_{x=m} \cong \frac{y_{m-2} - 4y_{m-1} + 6y_m - 4y_{m+1} + y_{m+2}}{h^4} \quad (5.5)$$

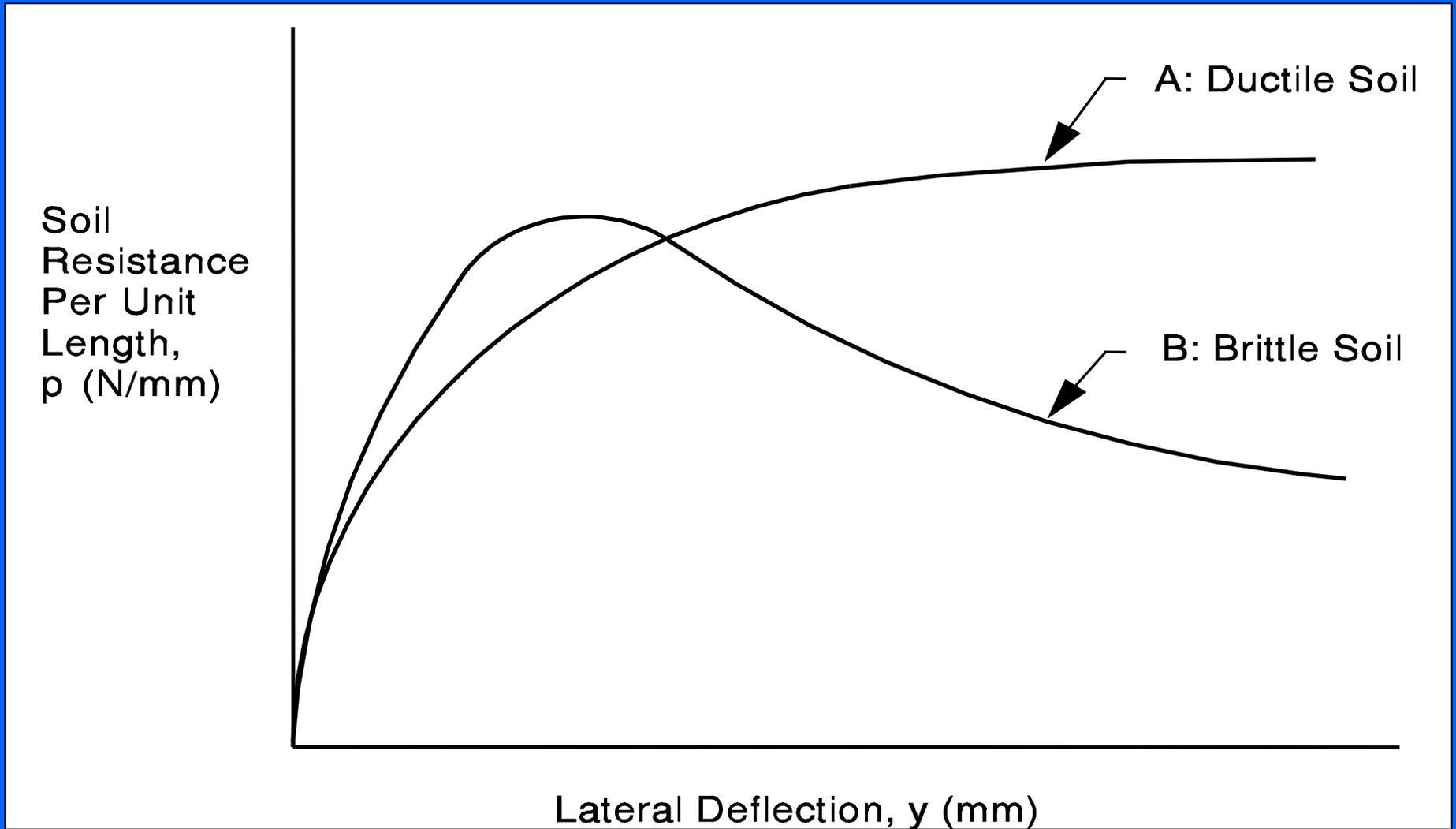


Figure 9.45 Typical  $p$ - $y$  Curves for Ductile and Brittle Soil (after Coduto, 1994)

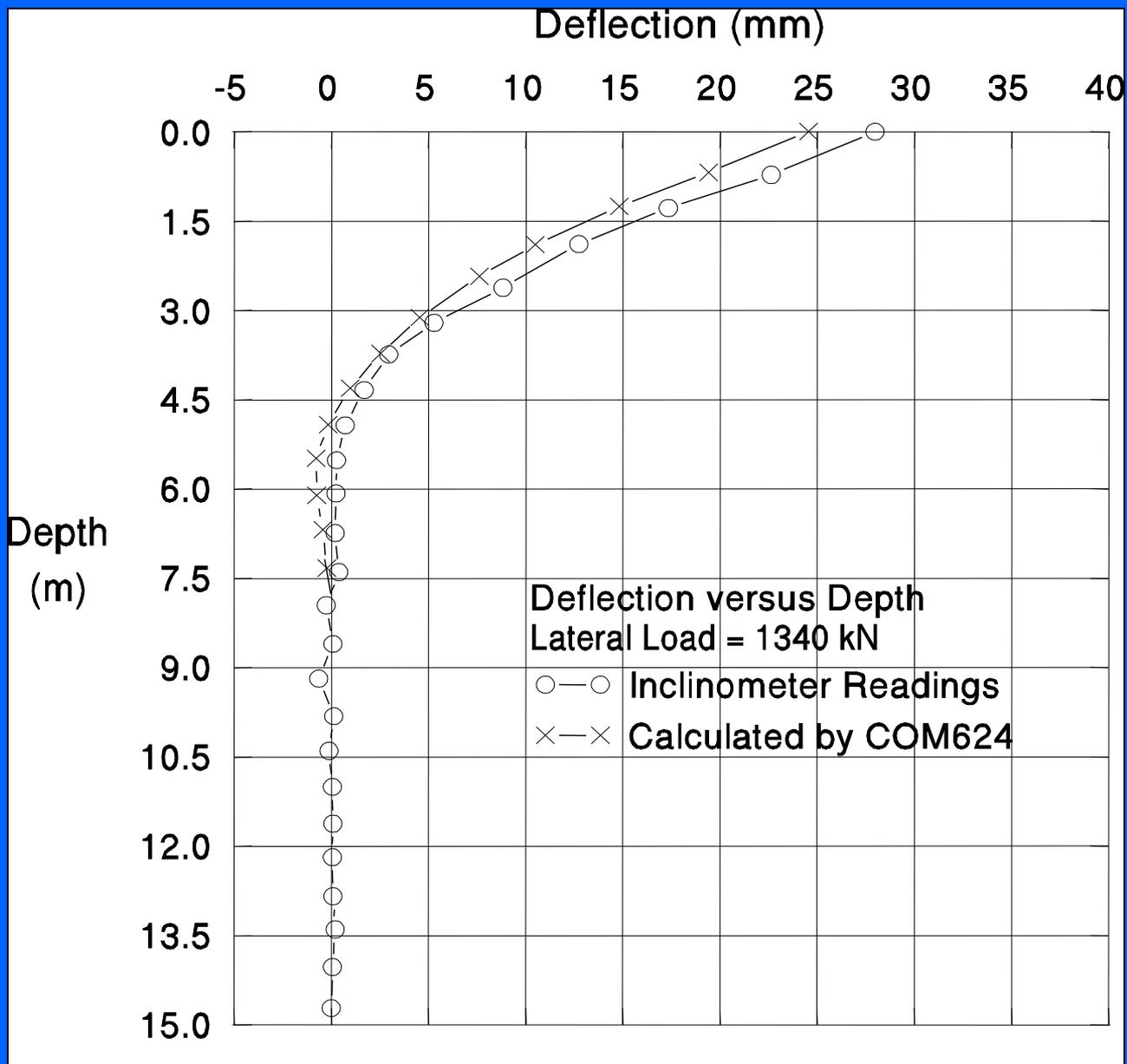


Figure 9.47 Comparison of Measured and COM624P Predicted Load-Deflection Behavior versus Depth (after Kyfor *et al.* 1992)

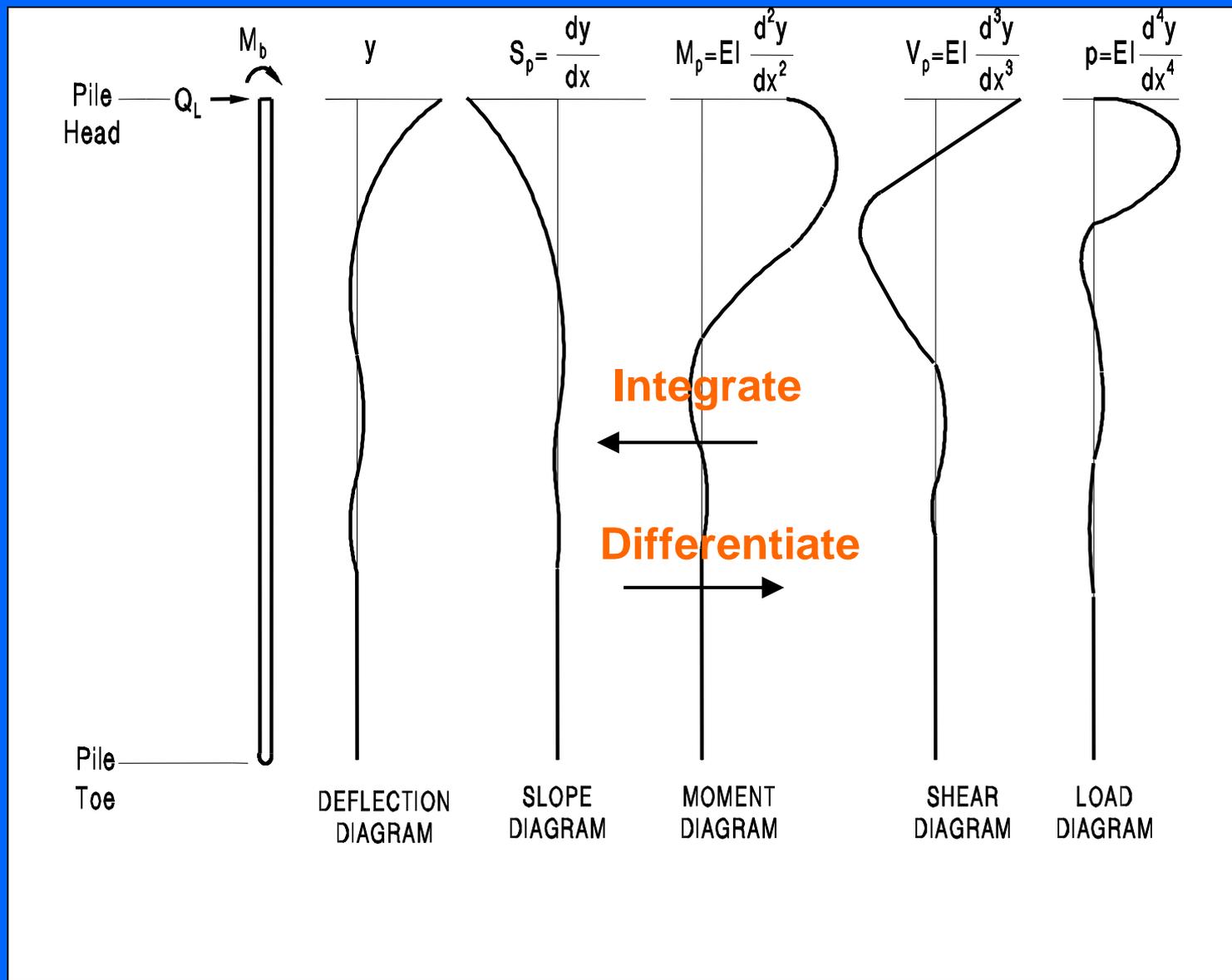


Figure 9.36 Graphical Presentation of LPILE Results (Reese, *et al.* 2000)

**ANY QUESTIONS ?**

